

## Question #1 ( Solution)

(a)

$$\left. \begin{aligned} I_{REF} = I_{D1} &= \frac{\mu_n C_{ox}}{2} \left(\frac{W}{L}\right)_1 (V_{GS1} - V_{T0n})^2 \\ I_2 = I_{D2} &= \frac{\mu_n C_{ox}}{2} \left(\frac{W}{L}\right)_2 (V_{GS2} - V_{T0n})^2 \\ V_{GS1} &= V_{GS2} \end{aligned} \right\} \Rightarrow I_2 = \frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1} I_{REF} = 2I_{REF}$$

$$I_{D4} = I_2 = 2I_{REF}$$

$$\left. \begin{aligned} 2I_{REF} = I_{D4} &= \frac{\mu_p C_{ox}}{2} \left(\frac{W}{L}\right)_4 (|V_{GS4}| - |V_{T0P}|)^2 \\ I_{D5} &= \frac{\mu_p C_{ox}}{2} \left(\frac{W}{L}\right)_5 (|V_{GS5}| - |V_{T0P}|)^2 \\ I_{D6} &= \frac{\mu_p C_{ox}}{2} \left(\frac{W}{L}\right)_6 (|V_{GS6}| - |V_{T0P}|)^2 \\ I_{D7} &= \frac{\mu_p C_{ox}}{2} \left(\frac{W}{L}\right)_7 (|V_{GS7}| - |V_{T0P}|)^2 \\ I_{D8} &= \frac{\mu_p C_{ox}}{2} \left(\frac{W}{L}\right)_8 (|V_{GS8}| - |V_{T0P}|)^2 \\ V_{GS4} &= V_{GS5} = V_{GS6} = V_{GS7} = V_{GS8} \end{aligned} \right\} \Rightarrow I_{D5} = I_{D6} = I_{D7} = I_{D8} = \frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_4} I_{D4} = 2I_{REF}$$

$$\Rightarrow \begin{cases} I_9 = I_{D5} + I_{D6} = 4I_{REF} \\ I_{10} = I_{D7} + I_{D8} = 4I_{REF} \end{cases}$$

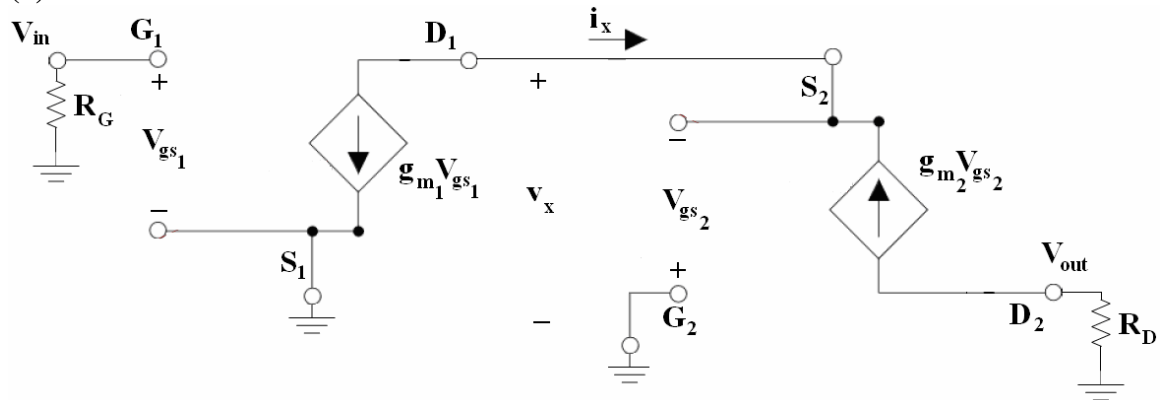
(b)

$$\left. \begin{aligned} I_{REF} = I_{D1} &= \frac{\mu_n C_{ox}}{2} \left(\frac{W}{L}\right)_1 (V_{GS1} - V_{T0n})^2 \\ I_3 = I_{D3} &= \frac{\mu_n C_{ox}}{2} \left(\frac{W}{L}\right)_3 (V_{GS2} - V_{T0n})^2 \\ V_{GS1} &= V_{GS3} \\ I_3 &= I_9 + I_{10} = 8I_{REF} \end{aligned} \right\} \Rightarrow I_3 = 8I_{REF} = \frac{\left(\frac{W}{L}\right)_3}{\left(\frac{W}{L}\right)_1} I_{REF}$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = 8\left(\frac{W}{L}\right)_1$$

**Question #2 ( Solution)**

(a)



**T model can be also used for M2.**

(b)

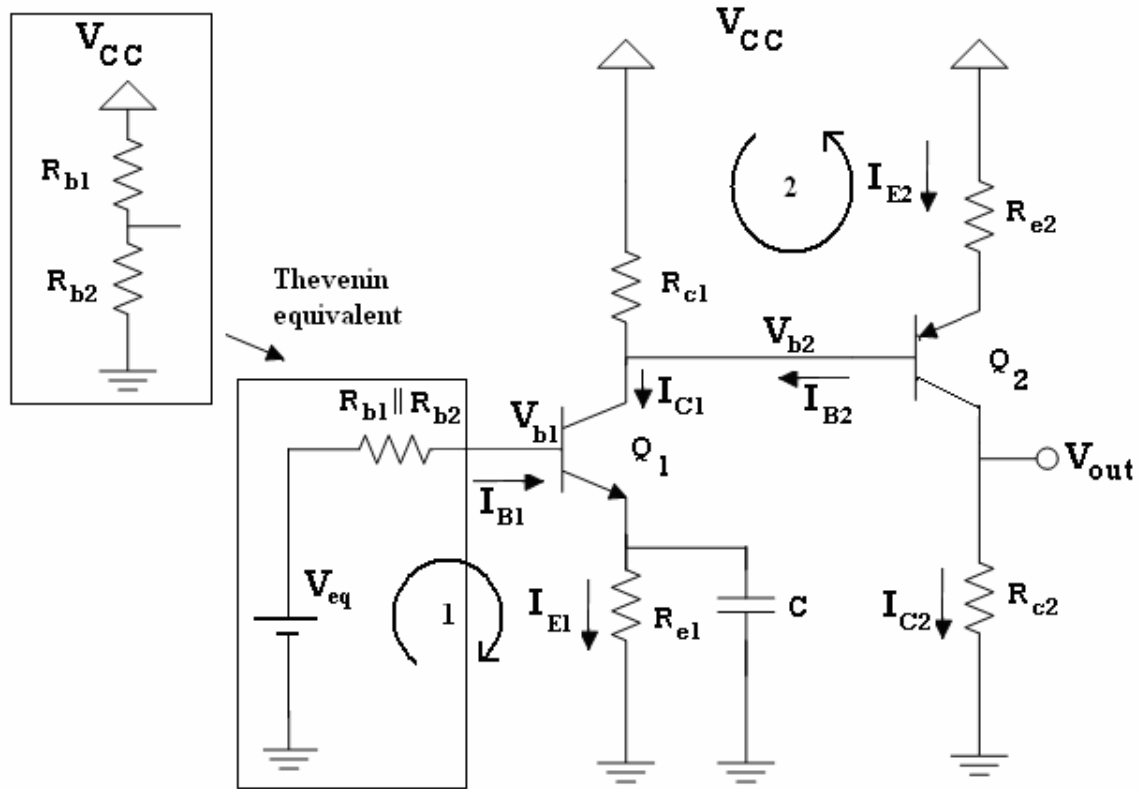
$$\left. \begin{aligned} R_{eq} &= \frac{v_X}{i_X} \\ V_{gs2} &= -v_X \\ i_X &= -g_{m2}V_{gs2} = g_{m2}v_X \end{aligned} \right\} \Rightarrow R_{eq} = \frac{1}{g_{m2}}$$

(c)

$$\left. \begin{aligned} i_{D1} &= g_{m1}V_{gs1} = g_{m1}V_{in} \\ V_{gs2} &= i_{D1}R_{eq} \\ V_{out} &= -i_{D2}R_D = -g_{m2}V_{gs2}R_D \end{aligned} \right\} \Rightarrow V_{out} = -g_{m1}R_DV_{in}$$

$$\Rightarrow A_v = \frac{V_{out}}{V_{in}} = -g_{m1}R_D$$

**Question #3 ( Solution)**



(a)

$$V_{eq} = \frac{R_{b2}}{R_{b2} + R_{b1}} V_{cc} = \frac{30}{30 + 70} 10 = 3V$$

$$R_{b1} \parallel R_{b2} = 30 \parallel 70 = 21K\Omega$$

Assume  $Q_1$  is operating in active mode and write the KVL at loop 1

$$V_{eq} - (R_{b1} \parallel R_{b2}) I_{B1} - V_{BE1(on)} - R_{e1} I_{E1} = 0$$

$$\Rightarrow 3 - 21 \times I_{B1} - 0.7 - 2 I_{B1} (100 + 1) = 0$$

$$\Rightarrow I_{B1} = 0.0103mA$$

$$\Rightarrow I_{C1} = 100 \times I_{B1} = 1.03mA$$

$$V_{B1} = V_{eq} - (R_{B1} \parallel R_{B2}) I_{B1} = 2.78V$$

$$V_{C1} = V_{B2} \quad V_{C1} - V_{B1} \geq 0 \quad \text{Active mode}$$

Assume  $Q_2$  is operating in active mode and write the KVL at loop 2

$$-(I_{C1} - I_{B2})R_{C1} + V_{BE2(on)} + I_{E2}R_{e2} = 0$$

$$\Rightarrow -(1.03 - I_{B2})4 + 0.7 + 3(100 + 1)I_{B2} = 0$$

$$\Rightarrow I_{B2} = 0.011mA$$

$$\Rightarrow I_{C2} = 100 \times I_{B2} = 1.1mA$$

$$V_{B2} = V_{CC} - R_{C1}(I_{C1} - I_{B2}) = 5.924V$$

$$V_{C2} = R_{C2}I_{C2}$$

$$V_{B2} - V_{C2} \geq 0 \quad \text{Active mode}$$

$$V_{CE1} = V_{B2} - R_{e1}I_{E1} = 5.924 - 2.08 = 3.844V \Rightarrow \text{Again we can conclude that } Q_1 \text{ is in active mode}$$

$$|V_{CE2}| = V_{B2} + 0.7 - R_{c2}I_{C2} = 6.624 - 4.4 = 2.224V \Rightarrow \text{Again we can conclude that } Q_2 \text{ is in active mode}$$

**(b)**

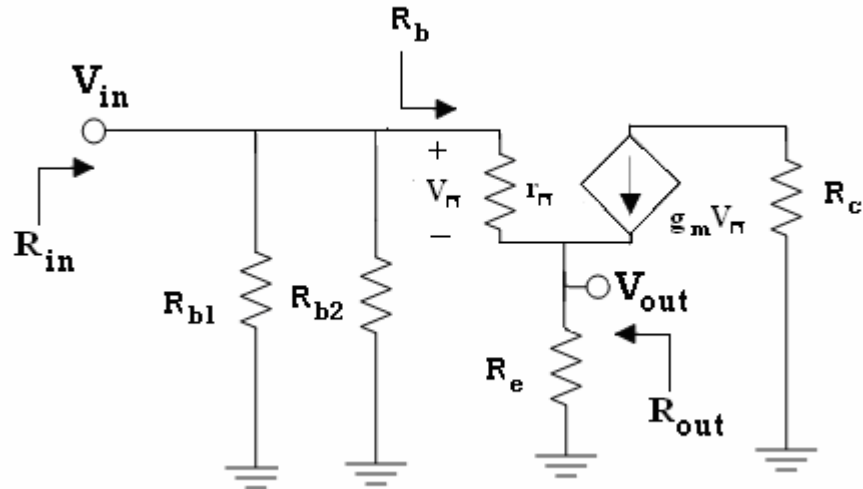
*In order for  $Q_2$  to turn off  $|V_{CE}|$  must equal  $|V_{CE(sat)}|$*

$$6.624 - R_{c2}I_{C2} = 6.624 - 1.1R_{c2} = 0$$

$$R_{c2} = 6.02 \text{ K}\Omega$$

**Question #4 ( Question)**

(a)



(b)

$$R_b = r_\pi + (1 + \beta)R_e$$

$$i_b = \frac{v_{in}}{R_b} = \frac{v_{in}}{r_\pi + (1 + \beta)R_e}$$

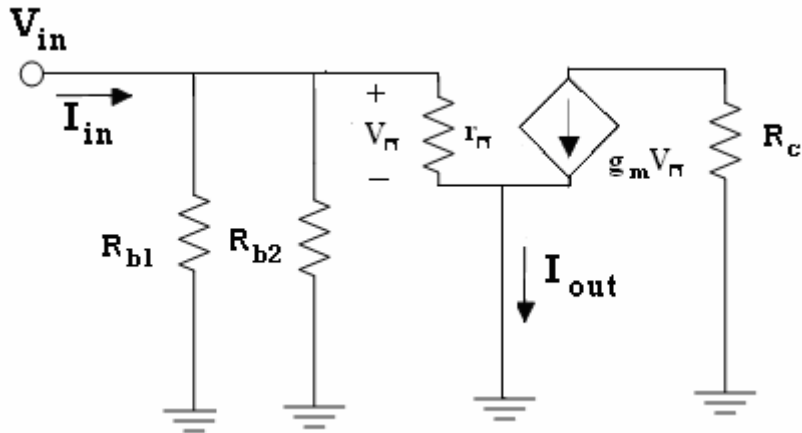
$$i_c = \beta i_b = \frac{v_{in}}{\frac{r_\pi}{\beta} + (1 + \frac{1}{\beta})R_e} = \frac{v_{in}}{r_e + (1 + \frac{1}{\beta})R_e}$$

$$i_e = \frac{\beta + 1}{\beta} i_c$$

$$v_{out} = i_e R_e = \frac{\beta + 1}{\beta} \frac{R_e}{r_e + (1 + \frac{1}{\beta})R_e} v_{in}$$

$$A_v = \frac{v_{out}}{v_{in}} = \frac{\beta + 1}{\beta} \frac{R_e}{r_e + (1 + \frac{1}{\beta})R_e} \cong \frac{R_e}{R_e + r_e}$$

(c) Short circuit the output to find  $A_{IS}$



$$i_{in} = \frac{v_{in}}{R_{b1} \parallel R_{b2}} + \frac{v_{in}}{r_{\pi}}$$

$$i_{out} = \frac{v_{in}}{\frac{r_{\pi}}{1 + \beta}} = \frac{v_{in}}{r_e}$$

$$\Rightarrow A_{IS} = \frac{i_{out}}{i_{in}} = \frac{\frac{1}{r_e}}{\frac{1}{(\beta + 1)r_e} + \frac{1}{R_{b1} \parallel R_{b2}}} = \frac{(\beta + 1)(R_{b1} \parallel R_{b2})}{(\beta + 1)r_e + R_{b1} \parallel R_{b2}}$$

(d)

$$R_{in} = R_{b1} \parallel R_{b2} \parallel R_b = R_{b1} \parallel R_{b2} \parallel (r_{\pi} + (1 + \beta)R_e)$$

$$R_{out} = \frac{v_{out}}{i_{out}} \Big|_{v_{in}=0} = R_e \parallel r_e$$