# Midterm # 2

## Question 1

a) For $M_2$: PMOS

$$\begin{cases} V_{DS_2} = V_{GS_2} = V_{ref} - V_{cc} = 3-5 = -2 < +V_{tp} = -1 \rightarrow \text{Triode / Satura[tion]} \\ V_{DS_2} < V_{GS_2} - V_{tp} \Rightarrow 0 < -(-1) = 1 \checkmark \rightarrow \text{Saturation mo[de]} \end{cases}$$

$$I_{ref} = \frac{1}{2} k'_p \frac{W_2}{L_2} (V_{GS_2} - V_{tp})^2 = 100\times10^{-6} (-2+1)^2 = 0.1 \text{ m[A]}$$

$$R_{ref} = \frac{V_{ref}}{I_{ref}} = \frac{3}{0.1} = 30 \text{ k}\Omega$$

$$\frac{W_3}{L_3} = 10 \frac{W_2}{L_2}, \quad V_{GS3} = V_{GS2}$$

$V_{DS3} = 3-5 = -2 < \overbrace{V_{GS3} - V_{tp}}^{3-(-1)=4}$ 

$V_{GS3} = 3-5 = -2 < V_{tp} = -1$ 

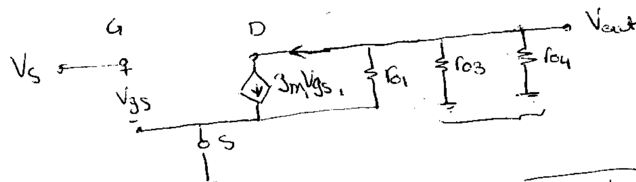
} saturation mode

$$\Rightarrow I_{D3} = \frac{1}{2} k'_p \overbrace{\frac{W_3}{L_3}}^{10\frac{W_2}{L_2}} (V_{GS2} - V_{tp})^2 = 10 \, I_{ref} = 1 \text{ m[A]}$$

$$I_{D4} = 10 \, I_{ref} = 1 \text{ mA}$$

$$I_{D_1} = I_{D3} + I_{D4} = 2 \text{ mA}$$

b)



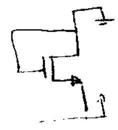

$$g_{m_1} = \sqrt{2k'_n} \sqrt{\frac{W_1}{L_1}} \sqrt{1+\lambda V_{DS_1}} \sqrt{I_{D_1}} = \sqrt{2\times200\times10^{-6}(1+0.05\times3)\,2\times10^{-3}} = 0.95[\ldots]$$

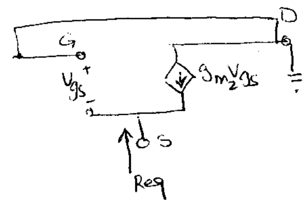
$$r_{o_1} = \frac{1}{\lambda I_{D_1}} = \frac{1}{0.05\times2\times10^{-3}} = 10 \text{ k}\Omega \qquad r_{o3} = r_{o4} = \frac{1}{\lambda I_{D3}} = \frac{1}{\lambda \frac{I_{D_1}}{2}} = 2 r_{o_1} = [\ldots]$$

c) $$V_{out} = -g_m \underbrace{V_{gs_1}}_{V_s} \Big(r_{o1} \,\|\, \underbrace{\overbrace{(2r_{o_1} \| 2r_{o_1})}^{r_{o_1}}}_{r_{o1}/2}\Big), \quad A_v = -g_m \frac{r_{o1}}{2} = -4.795 \text{ V/V}$$

# Question #2

a)

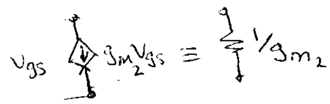





$V_{DS} = V_{gs}$

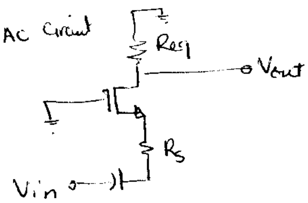
$\Downarrow$

source absorption

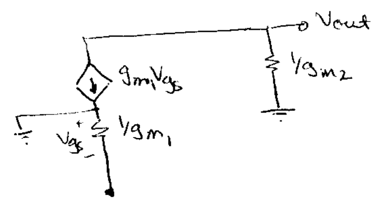



$$Req = \frac{1}{g_{m2}}$$

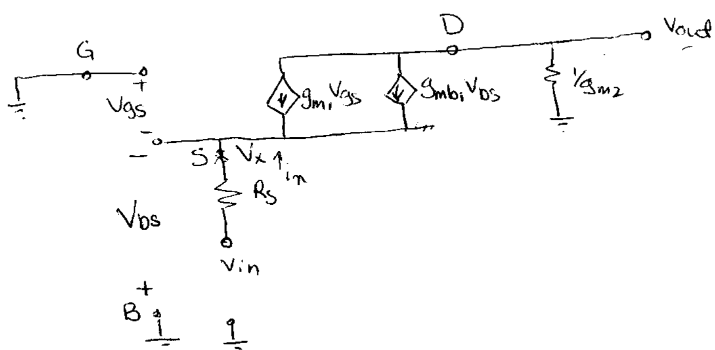
b) AC circuit




$\Rightarrow$ CGA




including body effect hybrid-$\pi$ model



c)

$$v_{out} = -(g_{m_1} V_{gs_1} + g_{mb_1} V_{bs_1}) \frac{1}{g_{m2}} \qquad V_{gs_1} = V_{bs_1}$$

$$V_x = -V_{bs_1} = -V_{gs_1}$$

$$A_v = \frac{V_{out}}{V_x} = +(g_{m_1} + g_{mb_1}) \frac{1}{g_{m2}}$$

$$A_{vout} = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_x} \cdot \frac{V_x}{V_{in}}$$

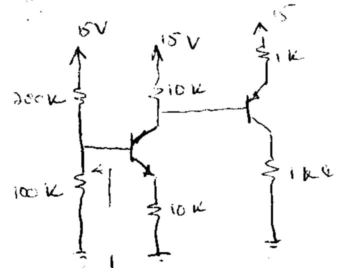
To Find $\frac{V_x}{V_{in}}$, we need to know Rin

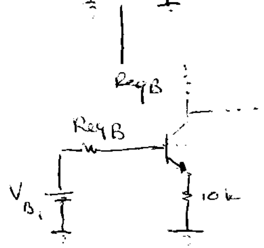
$$R_{in} = \frac{V_x}{i_x} = \frac{V_x}{-(g_{m_1} V_{gs_1} + g_{mb_1} V_{bs_1})} = \frac{1}{g_{m_1} + g_{mb_1}}$$

$$\frac{V_x}{V_{in}} = \frac{R_{in}}{R_s + R_{in}} = \frac{\frac{1}{g_{m_1} + g_{mb_1}}}{R_s + \frac{1}{g_{m_1} + g_{mb_1}}} = \frac{1}{R_s(g_{m_1} + g_{mb_1}) + 1}$$

$$A_{vout} = +(g_{m_1} + g_{mb_1}) \frac{1}{g_{m_2}} \cdot \frac{1}{RS(g_{m_1} + g_{mb_1}) + 1}$$

## Question 3




(1)

a) ∘ Assume active mode operation for both transistor and verify this later

$$V_{BB} = \frac{15}{100+200} \times 100 = 5\ V$$

$$R_{eqB} = 200 \| 100 = 66.7\ k\Omega$$

(1) $5 - R_{eqB}\, I_{B_1} - 0.7 - 10\underbrace{(\beta+1) I_{B_1}}_{I_{E_1}} = 0$

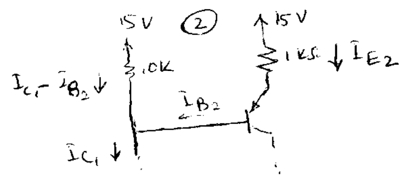
$4.3 - I_{B_1}(66.7 + 10 \times 101) = 0$

$\Rightarrow I_{B_1} = \dfrac{4.3}{66.7 + 10 \times 101} = \dfrac{4.3\ mA}{1076.7}$ $0.4\ mA$

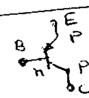
$I_{E_1} = 101\, I_{B_1} = 0.4034\ mA$

$V_{E_1} = 10\, I_{E_1} = 4.034\ V$

$\boxed{V_{B_1} = V_{E_1} + 0.7 = 4.734}$




b) 




$V_{BCsat} = V_{ECsat} - V_{EB}$

$V_{BCsat} = 0.3 - 0.7$
$= -0.4$ Volt

For active mode operation:

$\underbrace{V_{B2}}_{V_{C_1}} - \underbrace{V_{C2}}_{R_C 2.96} > -0.4$

$11.3 - R_C\, 2.96 > -0.4$

$R_C < \dfrac{11.7}{2.96} = 3.953\ k\Omega$

(2) $15 - 10(I_{C_1} - I_{B_2}) = 15 - 1\, I_{E2} - 0.7$

$-10(\alpha I_{E_1} - I_{B_2}) = -101\, I_{B2} - 0.7$

$10\left(\frac{100}{101} 0.4034 - I_{B_2}\right) = +101\, I_{B_2} + 0.7$

$10 \times \frac{100}{101} \times 0.4034 - 0.7 = 111\, I_{B_2}$ $\Rightarrow$ $I_{B_2} = 0.0297\ mA$

$\Rightarrow \boxed{I_{E2} = 2.99\ mA}$

$\rightarrow V_{E2} = 15 - I_{E2} 1 = 12.003\ V$ $\Rightarrow \boxed{V_{C_1} = 11.3}$ $V \Rightarrow$ $V_{CB_1} = 6.57V$, $V_{CB_1} > 0$ Reverse bias

$V_{C2} = \alpha I_{E2}\ 1k = 2.96\ mA\ 1k = 2.96\ V$

$\Rightarrow$ Active mode operation for both is verified $\Rightarrow |V_{BC2} = 8.34 > 0$ Reverse bias

## Question 4

a) $-2 - R_B I_B - 0.7 - R_E (\beta+1) I_B - 5 = 0$

$7.7 = (100k + 101 \times 4k) I_B \Rightarrow I_B = \frac{7.7}{504k} =$

$I_E = (\beta+1) I_B = 1.26 \text{ mA}$

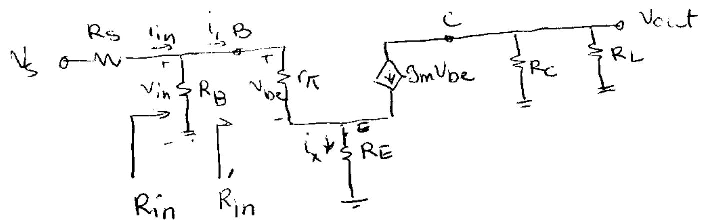
$I_C = \alpha I_E = 1.25 \text{ mA}$

$V_E = -5 + I_E R_E = 0.05 \text{ V}$

$V_B = 0.7 + V_E = 0.75 \text{ V}$

$V_C = 5 - I_C \overset{1.7}{R_C} = 2.88 \text{ V}$

b)




$V_{in} = v_{be} + i_x R_E$

$i_x = \underbrace{\frac{v_{be}}{r_\pi}}_{i_1} + g_m v_{be} R_E = v_{be} \left(\frac{1}{r_\pi} + g_m\right) = \frac{v_{be}}{r_\pi} \quad (\beta+1) R_E$

$v_{in} = v_{be} + \frac{v_{be}}{r_\pi} R_E (1+\beta) = \overbrace{\frac{v_{be}}{r_\pi}}^{i_1} \left[ r_\pi + R_E (1+\beta) \right]$

$R'_{in} = \frac{v_{in}}{i_1} = \left[ r_\pi + R_E (1+\beta) \right]$, $r_\pi = \frac{V_T}{I_B} = 19.845$

$R_{in} = R_B \parallel R'_{in} = R_B \parallel \left[ r_\pi + R_E (1+\beta) \right] = 80.2 \text{ k}\Omega$

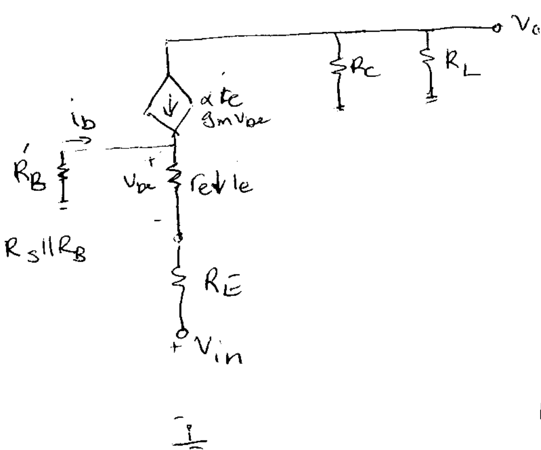
c) $A_v = ? = \frac{v_{out}}{v_S} = \frac{v_{out}}{v_{in}} \cdot \frac{v_{in}}{v_S} = \frac{v_{out}}{v_{in}} \cdot \frac{R_{in}}{R_S + R_{in}}$

$$\begin{cases} v_{out} = -g_m v_{be} (R_C \| R_L) \\ v_{in} = \frac{v_{be}}{r_\pi} \left[ r_\pi + R_E (1+\beta) \right] \end{cases} \Rightarrow \frac{v_{out}}{v_{in}} = \frac{-g_m r_\pi (R_C \| R_L)}{r_\pi + (\beta+1) R_E} = -0.155$$

$$\frac{v_{out}}{v_{in}} = \frac{-\beta \, (R_C \| R_L)}{r_\pi + (\beta+1) R_E}$$

$$A_v = \frac{-\beta (R_C \| R_L)}{r_\pi + (\beta+1) R_E} \cdot \frac{R_{in}}{R_S + R_{in}} = -0.138 \text{ V/V}$$

d) source at the Emitter $\Rightarrow$ CBA




$R_B' = R_S \| R_B$

$$v_{out} = -\underbrace{g_m v_{be}}_{\alpha i_e} (R_C \| R_L)$$

$$v_{in} = -(r_e + R_E) i_e - i_b R_B'$$

$$= \left[ -(r_e + R_E) - \frac{R_B'}{\beta+1} \right] i_e$$

$$A_v = \frac{-\alpha \cancel{i_e} (R_C \| R_L)}{-\cancel{i_e} \left[ r_e + R_E + \frac{R_B'}{\beta+1} \right]}$$

$$A_v = \frac{\beta}{\beta+1} \cdot \frac{R_C \| R_L}{(r_e + R_E) + \frac{R_B'}{\beta+1}}$$