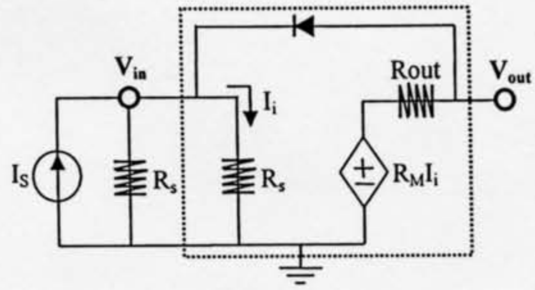
**Question#1 (9 pts.):**

(a) [2 pts.]
Correct Diagram:




*[-1 for incorrect source]*
*[-0.5 for incorrect label]*
*[0 for simply redrawing the given diagram]*

(b) [2 pts.]
The diode is OFF.

$Vin = Is \bullet Rs||Ri$
$Vout = R_M \bullet Ii$
$Ii = [Rs/(Rs+Ri)] \bullet Is$
*[-1 for incorrect Vin, Vout, or Ii]*

$:.\ Vout/Vin = R_M \bullet [Rs/(Rs+Ri)] \bullet [1/(Rs||Ri)]$
$= R_M \bullet [Rs/(Rs+Ri)] \bullet [(Rs+Ri)/(Rs \bullet Ri)]$
$= R_M/Ri$
$= 400/100$
$= 4$
*[-1 for incorrect Vout/Vin expression]*
*[-0.5 for mathematical error]*

(c) [2 pts.]
Clipping occurs when (Vout-Vin=0.7V)
*[1 point for understanding this]*

$Vout = R_M \bullet [Rs/(Rs+Ri)] \bullet Is$
$Vin = [(Rs \bullet Ri)/(Rs+Ri)] \bullet Is$
$:.\ Is \bullet [(R_M \bullet Rs)-(Rs \bullet Ri)/(Rs+Ri)] = 0.7$
$:.\ Is = 0.7(1.2k+100)/1.2K(400-100)$
$= 2.53mA$
*[1 point for these calculations]*

(d) [1 pt.]

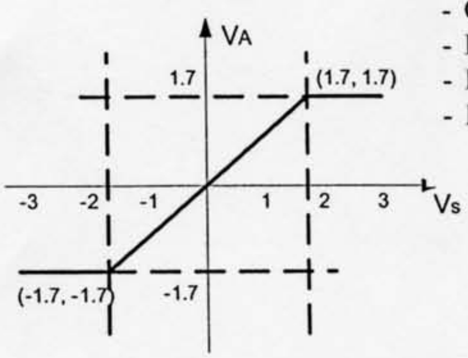
In the linear region, the diode is OFF.
:. Rout = Ro =150Ω.
*[No part marks]*

(e) [2 pts.]
$I_L = (R_M \cdot Ii)/(Ro+R_L)$
$Ii = [Rs/(Rs+Rin)] \cdot Is$
:. $I_L/Is = (R_M \cdot Rs)/[(Ro+R_L) \cdot (Rs+Rin)]$
$= (400 \cdot 1.2K) / (200+1.3K)$
$= 1.846$ A/A

*[-1 for incorrect $I_L$, Ii, or $I_L$/Is expressions;*
*-1.5 for 2 out 3 incorrect]*
*[-1 for taking Vout from the previous parts and dividing by $R_L$. That is incorrect]*
*[-0.5 for mathematical errors]*

[Q2 Solution]:

a) [2 pts]




- Curve doesn't cross origin, -0.5 pts;
- Do not mark $V_A$ axis correctly, -0.5 pts;
- Do not mark $V_S$ axis correctly, -0.5 pts;
- Do not show voltage limiting on both sides, -0.5 pts;

b) [4pts] Since the DC component of Vs is 0, so both D1 and D2 are OFF by inspection, hence $I_{D1} = I_{D2} = 0$;

By inspection, D4 and D5 cannot be both ON.

Assume D3, D4 and D6 ON, D5 OFF.

[Find out $I_{D1} = I_{D2} = 0$ and made some assumptions, should get 0.5 pts;]

$$I_{D3} = \frac{3-(V_B+0.7)}{500} = \frac{2.3-V_B}{500}$$

$$I_{D6} = \frac{3-(V_C+0.7)}{1k} = \frac{2.3-V_c}{1k}$$

$$V_B = V_C + I_{D4} \times 100 + 0.7$$

By KCL:

$$I_{D3} = I_{D4} + I_1 = I_{D4} + 0.68m$$

$$I_{D4} + I_{D6} = I_2 = 10m$$

[Write these two sets equations right, got another 3 pts;]

Solve the above equations and got:

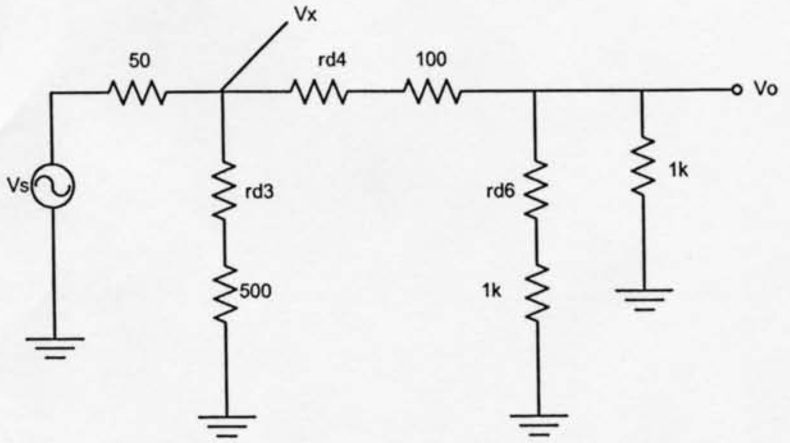
$V_B = -0.84V$ $\quad V_C = -2.1V$ $\quad V_C - V_B = -1.26V$ *so D5 is OFF.*

$I_{D3} = 6.8mA$ $\quad I_{D4} = 5.6mA$ $\quad I_{D5} = 0$ *and* $I_{D6} = 4.4mA$

[Find out the final answers correct, got another 0.5 pts, so total 4 pts;]

[Note: Failed to justify the assumption, -0.5 pts;

c) [2 pts]




The small-signal must be consistent with the results in their part b). If due to the mistakes of part b that make part c any easier then expected, -1 pts;

For any single mistake in this part, -0.5 pts;

d)

[2 pts]

$$\frac{vo}{vs} = \frac{vo}{vx} \times \frac{vx}{vs}$$

$$\frac{vo}{vx} = \frac{1k // (rd6 + 1k)}{rd4 + 100 + 1k // (rd6 + 1k)} \qquad \text{1 pts}$$

$$\frac{vx}{vs} = \frac{(rd3 + 500) // (rd4 + 100 + 1k // (rd6 + 1k))}{50 + (rd3 + 500) // (rd4 + 100 + 1k // (rd6 + 1k))} \qquad \text{1 pts}$$

e) [3 pts] For D4 D5 both OFF,

$|V_B - V_C| < 0.7V$ (*) 0.5 pts

$V_B = 3 - 500 * I_1 - 0.7$ (1) 0.5 pts

$V_C = 3 - 1k * I_2 - 0.7$ (2) 0.5 pts

Substitute (1), (2) into (*)

$|10 - 500 * I_1| < 0.7$

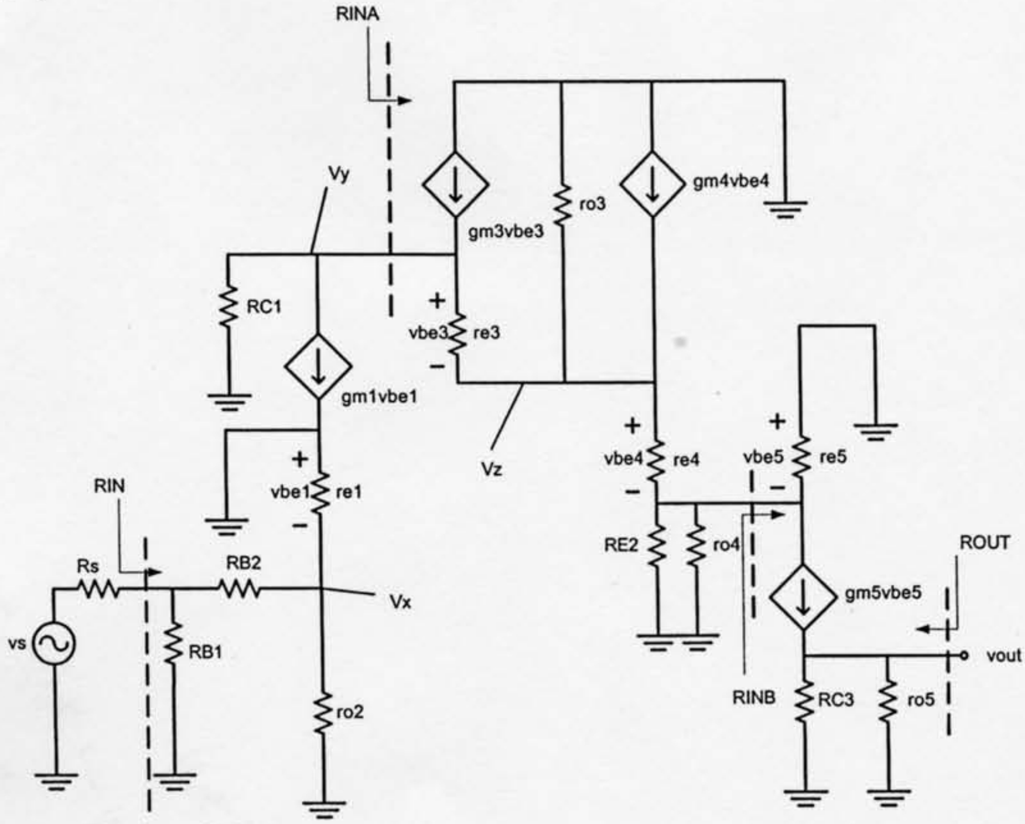
$18.6mA < I_1 < 21.4mA$ 1.5 pts

If only got half of the answer, should get 1.5 pts in total; And if include 18.6mA and 21.4mA as the answer, -0.5 pts;

[Q3 Solution]:

a)

[5 pts]




For any mistake, - 0.5 pts;

b) $R_{IN} = R_{B1} \,//\, (R_{B2} + r_{o2}//r_{e1})$ [1 pts]

$R_{INA} = (\beta+1)\ [r_{e3} + r_{o3} \,//\, ((\beta+1)\ (r_{e4} + r_{o4}//R_{E2}//R_{INB}))]$ [1 pts]

$R_{INB} = (\beta+1)\ r_{e5}$ [1 pts]

$R_{out} = r_{o5} \,//\, R_{C3}$ [1 pts]

For any mistake in part a) that makes part b) easier than expected, -0.5 pts.

c) [4 pts] 1 pts for each equation, -0.5 pts for partial answers;

$$\frac{vx}{vs} = \frac{ro//re1}{RB2 + ro2//re1} \times \frac{Rin}{Rs + Rin}$$

$$\frac{vy}{vx} = gm1 \cdot (Rc1//RINA)$$

$$\frac{vz}{vy} = \frac{RE2//ro4//RINB}{re4 + RE2//ro4//RINB} \times \frac{ro3//[(\beta+1)[re4 + RE2//ro4//RINB]]}{re3 + ro3//[(\beta+1)[re4 + RE2//ro4//RINB]]}$$

$$\frac{vout}{vz} = -gm5 \cdot (Rc3//ro5)$$

Solution

(Q4)

$V_{DD} = 5V,\ \lambda = 0.02\,V^{-1},\ |V_{t_0}| = 1V$

a) $V_{t_1} = 1V$ (no Body effect) 0.5

$$V_{t_2} = V_{t_0} + \gamma\left(\sqrt{2\phi + V_{SB}} - \sqrt{2\phi}\right) = 1V + 0.75\left(\sqrt{2.4+1} - \sqrt{2.4}\right)$$ 0.5

$$= 1.221\ V$$

b) both FETs are in saturation, so...

$$I_3 = I_1 + I_2 = \tfrac{1}{2}k\tfrac{W}{L}\left(V_{GS_1} - V_{t_1}\right)^2\left(1+\lambda V_{DS}\right) + \tfrac{1}{2}k\tfrac{W}{L}\left(V_{GS_2} - V_{t_2}\right)^2$$ Eqn: 1.5pt

$$4mA = \tfrac{1}{2}k\tfrac{W}{L}\left[(3-1)^2(1+0.02(3)) + (3-1-1.221)^2\right]$$ -1 if neglect CLM or Body

$$k\tfrac{W}{L} = 1.6505\ mA/V^2$$ ans: 0.5pt (-0.5 for math error)

c) $I_1 = \tfrac{1}{2}k\tfrac{W}{L}\left((3-1)^2(1+0.02(3))\right)$

$= 3.5mA$

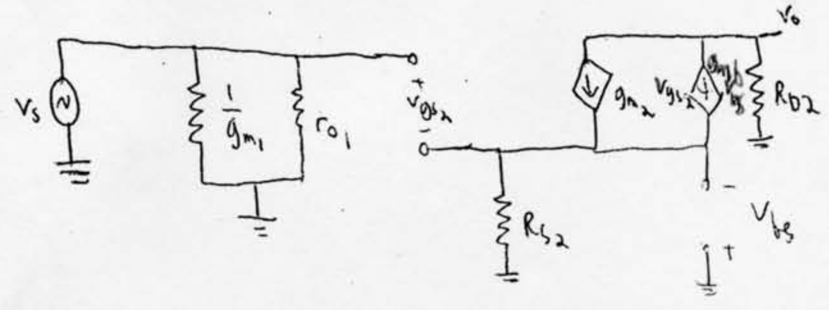
$I_2 = I_3 - I_1 = 0.5mA \Rightarrow R_{S2} = \dfrac{1-0}{0.5mA} = 2k\Omega$ ← 1pt

M2 sat mode requires $V_D > V_G - V_{t_2}$

$> 1.779$

so $R_{D2} < \dfrac{5-1.779}{0.5mA} = 6.442k\Omega$ ✓ 1pt (-0.5 if inequality sign is wrong)

d)




(-1 Forgetting Body)

2pts

(-0.5 if "$r_o$" alone)

1pt

F) $R_{in} = \dfrac{1}{g_{m_1}} \| r_{o_1}$

e) $v_{gs_2} = v_s - g_{m_2}v_{gs_2}R_{S2} - g_{mb}v_{bs_2}R_{S2}$ $\Rightarrow v_{gs_2} = \dfrac{v_s}{1+g_{m_2}R_{S2}}$ and $v_o = -g_{m_2}v_{gs_2}R_{D2}$

$-V_{bs_2} = V_S - V_{gs_2}$ $\dfrac{g_{mb}R_s}{1-g_{mb}R_s}$

$\dfrac{v_o}{v_s} = \dfrac{-R_{D2}}{\frac{1}{g_{m_2}} + R_{S2}}$ $= \dfrac{-g_{m_2}R_{D2}}{\ldots}$

See next page

2pts

1/4

e) $\frac{-v_o}{R_D} = \frac{v_s - v_{gs_2}}{R_{s_2}}$ ①

$\Rightarrow$ need $v_{gs}$ in terms of $v_s$!

$$v_{gs_2} = v_s - \left[g_{m_2} v_{gs_2} + g_{m_b} v_{bs}\right] R_s$$

$$= v_s - \left[g_{m_2} v_{gs_2} + g_{m_b}(v_{gs_2} - v_s)\right] R_s \quad (\text{using } -v_{bs} = v_s - v_{gs_2})$$

$$v_{gs_2}\left[1 + g_{m_2}R_s + g_{m_b}R_s\right] = v_s\left[1 + g_{m_b}R_s\right]$$

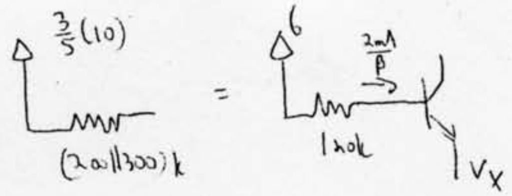
$$v_{gs_2} = v_s \frac{\left[1 + g_{m_b}R_s\right]}{\left[1 + g_{m_b}R_s + g_{m_2}R_s\right]}$$

$\Rightarrow$ ① $\therefore$

$$\boxed{\frac{v_o}{v_s} = -\frac{R_D}{R_{s_2}}\left(1 - \frac{1 + g_{m_b}R_{s_2}}{1 + g_{m_b}R_{s_2} + g_{m_2}R_{s_2}}\right)}$$

Q5

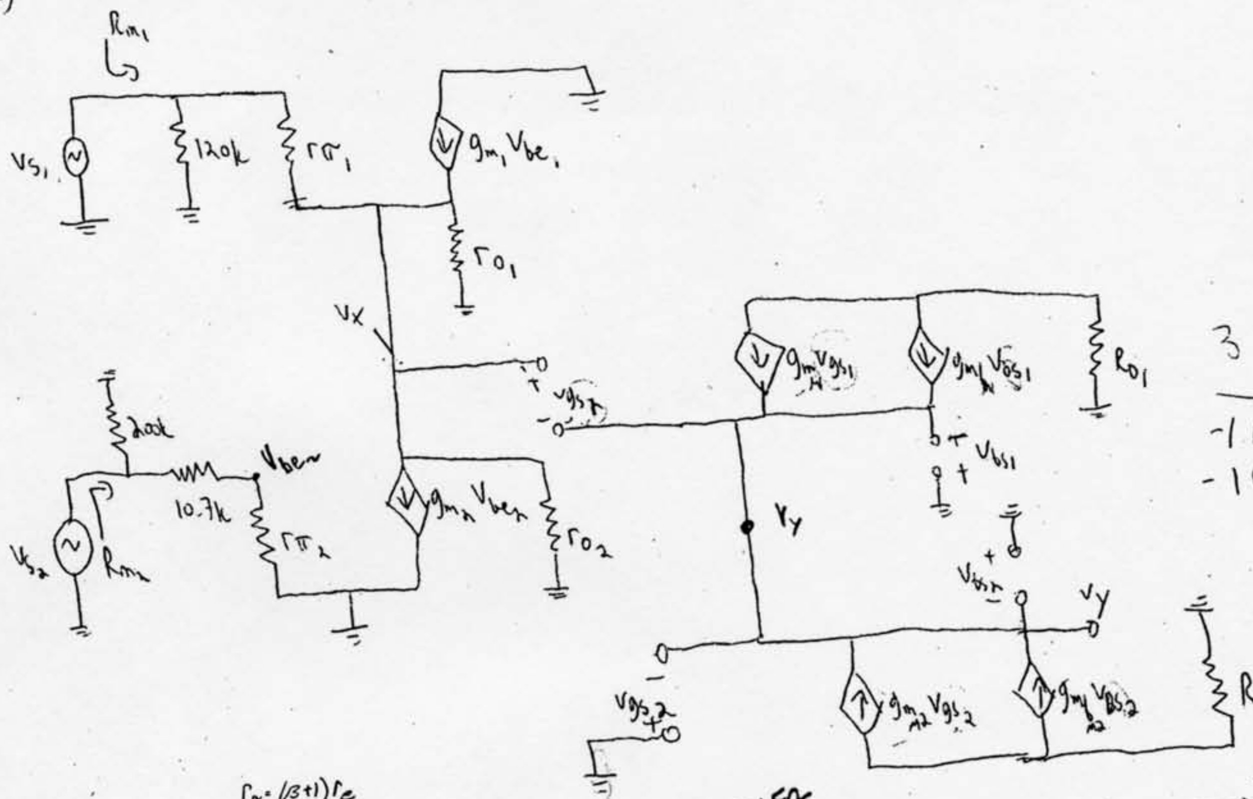
a) by Thévenin, input looks like




$$V_X = 6 - \frac{2mA}{99}(120k) - 0.7 = 2.876\ V$$ 2 pts

(−1 for incorrect Thévenin)

b)




3 pts

−1 pt for no $r_{o1}, r_{o2}$

−1 pt for no $V_{SB}$

c) $r_\pi = (\beta+1)r_e$

$120K \,//\, (\beta+1)(r_{e1} + r_{o1}//r_{o2})$

$$R_{in1} = 120k \| \left[ r_{\pi 1} + (\beta+1)(r_{o1}\|r_{o2}) \right]$$ 1 pT.

$$\frac{v_x}{v_{s1}} = \frac{(\beta+1)(r_{o1}\|r_{o2})}{(\beta+1)(r_{o1}\|r_{o2}) + r_{\pi 1}}$$ 2 pts.

d) $R_{in2} = 200k \| (10.7k + r_{\pi 2})$ 1 pT.

→ OVER

$$\frac{V_x}{V_{s2}} = \frac{V_x}{V_{be2}} \cdot \frac{V_{be2}}{V_{s2}}$$

$\Rightarrow V_{be2} = V_{s2}\frac{r_{\pi 2}}{r_{\pi 2}+10.7k}$ ; $g_{m2}V_{be2} = g_{m1}\underset{(-V_x)}{V_{be1}} - \frac{V_x}{r_{\pi 1}\|r_{o1}\|r_{o2}}$

$$\Rightarrow g_{m2} v_{s2}\frac{r_{\pi 2}}{r_{\pi 2}+10.7k} = -V_x\left[g_{m1} - \frac{1}{r_{\pi 1}\|r_{o1}\|r_{o2}}\right]$$

$$\frac{V_x}{V_{s2}} = \frac{\frac{g_{m2} r_{\pi 2}}{r_{\pi 2}+10.7k}}{\frac{1}{r_{\pi 1}\|r_{o1}\|r_{o2}} + g_{m1}}$$ 2 pts.

or → $\frac{-\beta\left[r_{e1}\|r_{o1}\|r_{o2}\right]}{r_{\pi 2}+10.7k}$

... if they ignore $V_{be1}$ take off 1 mark

e) $V_{gs1} = V_x - V_y$, $V_{gs2} = -V_y$, $V_{bs1} = -V_y$, $V_{bs2} = -V_y$

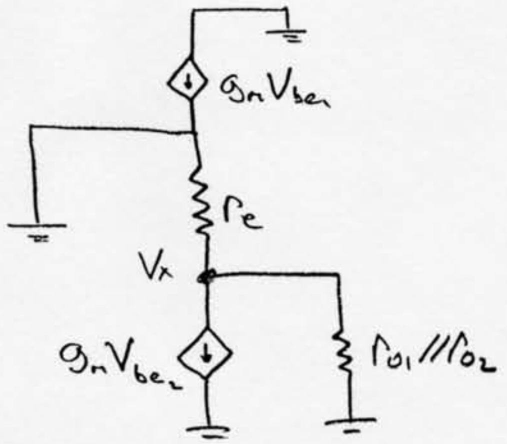
$g_{mM}(V_{gs1}+V_{gs2}) + g_{mbM}(V_{bs1}+V_{bs2}) = 0$; $g_{mM}(V_x - V_y - V_y) + g_{mbM}(-V_y - V_y)$

$V_y(2g_{mM} + 2g_{mbM}) = g_{mM}V_x$

$$\frac{V_y}{V_x} = \frac{g_{mM}}{2g_{mM}+2g_{mbM}} = \frac{1}{2\left(1+\frac{g_{mbM}}{g_{mM}}\right)}$$ 2 pts.

5d) From T-MODEL

$$V_{be_2} = V_{s_2} \frac{r_{\pi_2}}{r_{\pi_2} + 10.7k}$$




$$\frac{0 - V_x}{r_e} = g_m V_{be_2} + \frac{V_x}{r_{o1} // r_{o2}}$$

$$\frac{V_x}{V_{be_2}} = -g_m \left( r_{o1} // r_{o2} // r_e \right)$$

$$\frac{V_x}{V_{s_2}} = -g_{m_2} \left( r_{o1} // r_{o2} // r_e \right) \frac{r_{\pi_2}}{r_{\pi_2} + 10.7k}$$

**Question#6 (9 pts.):**

(a) [3 pts.]

$I_{REF} = (1.8\text{-}1.2)/12K = 50\mu A$ *[1 Point]*

Ioi may be estimated as follows:

:. $W_2/W_1 = 3\mu/1.2\mu = 2.5$

:. $Ioi = 2.5 \cdot 50\mu A = 125\mu A$ *[1 Point]*

$Roi = 1/(\lambda Ioi) = 1 / (0.02 \cdot 125\mu) = 400k\Omega$ *[1 Point]*

(b) [1 point]

The input resistance into a FET gate is $Rin = \infty$ *[No part marks]*

(c) [3 Pts]

*[Both Rout1 and Rout2 may be solved by symmetry/single-ended treatment. Each is worth 1.5 points. Partial marks are given for correct method but incorrect answer]*

$Rout1 = 2 \cdot (R_L/2 \parallel R_D)$ $\Omega$ *[1.5 Pts.]*

$Rout2 = 2 \cdot (Roi \parallel 1/gm)$ $\Omega$ *[1.5 Pts.]*

(d) [3 Pts.]

*[Both Vo1/Vin and Vo2/Vin may be solved by symmetry/single-ended treatment. Each is worth 1.5 points. Partial marks are given for correct method but incorrect answer]*

$Vo1/Vin = (V_{D1}\text{-}V_{D2})/Vin = -gm \cdot [R_L/2 \parallel R_D]$ V/V *[1.5 Pts.]*

$Vo2/Vin = (V_{S1}\text{-}V_{S2})/Vin = 1/(1/gm + Roi)$ V/V *[1.5 Pts.]*

Alternatively, one may use voltage divider to determine Vo2, using Vo1 as the input voltage. This gives the following answer:

$Vo2/Vin = (V_{S1}\text{-}V_{S2})/Vin = Roi / (1/gm + Roi)$ V/V *[Also acceptable for 1.5 Pts.]*