# McGill University <br> Faculty of Engineering <br> Department of Electrical and Computer Engineering ECSE-330A - Introduction to Electronics 

Examiner: Dr. David V. Plant;
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Date: Tuesday, December 21, 2004
Time: 2:00-5:00
Calculator: Faculty Standard

## Pertinent Information:

1) This is a closed-book examination, no notes permitted. There are 3 pages of equations provided at the back of the examination.
2) The examination consists of 6 problems; you must answer all 6 problems.
3) The examination is worth 70 total points
4) The examination consists of 10 pages, including this page and the equations pages; please ensure you have a COMPLETE examination paper.
5) Only the Faculty Standard Calculator is permitted.
6) Questions may be completed in any order, however ensure that you clearly identify which part of which question you are attempting.

## Do NOT turn in this exam with your exam booklet

## Question \#1 (9 pts.):

A transresistance amplifier with input resistance $100 \Omega$, output resistance $150 \Omega$, and open-circuit transresistance $400 \mathrm{~V} / \mathrm{A}$ is rectified by adding a feedback diode as shown below. A current source with a source resistance $1.2 \mathrm{k} \Omega$ is applied at the input. You may assume the 0.7 V -drop model for the diode.


For parts a), b), c) and d) there is no load attached (Vout is an open-circuit).
a) [2 pts.] Redraw the circuit and replace the amplifier with its equivalent circuit. (Note: Leave the diode as it is, do not replace it with its small signal model).
b) [2 pts.] Give the voltage gain Vout/Vin when the setup is in the linear region of operation (i.e. before clipping occurs at Vout).
c) [2 pts.] At what value of $I_{s}$ does clipping occur at Vout?
d) [1 pt.] Determine the output resistance, Rout, during linear operation. (Note: you may assume $\mathrm{I}_{\mathrm{S}}$ is open-circuited from the input to find Rout).

## For part (e), the diode has been disconnected.

e) $[2$ pts. $]$ A load, $\mathrm{R}_{\mathrm{L}}=50 \Omega$ has been added at the output as shown in the diagram. Determine the current gain, Ai , of the setup. (Hint: $\mathrm{Ai}=\mathrm{I}_{\mathrm{L}} / \mathrm{I}_{\mathrm{S}}$ ).

## Question \#2 (13 pts):

In the following circuit, all diodes are identical. Use the constant voltage drop model.

a) [2 pts.] Assume $\mathrm{V}_{\mathrm{S}}$ is a $\underline{\mathrm{DC}}$ source and sketch the voltage transfer curve of $V_{A}$ vs. $V_{S}$ for $V_{S}$ values from -3V DC to +3 V DC. Mark all relevant points on the plot clearly.

For parts b), c) and d), assume $\mathrm{Vs}=\boldsymbol{v}_{\mathrm{s}}$, a small-signal voltage source with 0 V DC. Use $I_{1}=0.68 \mathrm{~mA}$ and $R_{P}=100 \Omega$.
b) [4 pts.] Find the DC current passing through each diode in the circuit, ( $I_{D 1}$ through $I_{D 6}$ ) and also the DC voltage of $V_{B}$ and $V_{C}$. Clearly specify your assumptions and justify them.
c) [2 pts.] Draw the small-signal equivalent circuit. Do NOT calculate the diode small-signal resistances;
d) [2 pts.] Derive the expression of $v_{O} / v_{S}$. Do NOT attempt to simplify.

For part e), assume $\boldsymbol{R}_{P}=\mathbf{0}$.
e) $[3 \mathrm{pts}$.$] For what range of values of I_{1}$ are both D4 and D5 OFF?

## Question \#3 (13 pts)

In this circuit, all transistors are active and have the same $\beta$ value. You must decide where to include or neglect the Early Effect in your analysis. You may refer to indicated resistances as parts of your answer (for example, you may refer to $\mathrm{R}_{\mathrm{inB}}$ in your expression for $\mathrm{R}_{\mathrm{inA}}$ )

a) [5 pts.] Draw a small-signal model for this circuit.
b) [4 pts.] Find expressions for $R_{i n}, R_{\text {inA }}, R_{i n B}$ and $R_{\text {out }}$ in terms of the resistances and small-signal parameters of the transistors.
c) [4 pts.] Find expressions for the voltage gains $v_{x} / v_{s}, v_{y} / v_{x}, v_{z} / v_{y}$ and $v_{\text {out }} / v_{z}$ in terms of the resistances and small-signal parameters of the transistors.

## Question \#4 (10 pts)

Consider the circuit below. For all parts of this question, you must decide when to include CLM and the Body Effect for these FETs.


The following is known:

- $\lambda=0.02 \mathrm{~V}^{-1}$ and $\left|\mathrm{Vt}_{0}\right|=1 \mathrm{~V}$
- $\gamma=0.75 \mathrm{~V}^{1 / 2}$ and $\varphi=1.2 \mathrm{~V}$
- $\mathrm{V}_{\mathrm{G} 1}=3 \mathrm{~V}, \mathrm{~V}_{\mathrm{S} 2}=1 \mathrm{~V}$
- The power supply $V_{D D}=5 \mathrm{~V}$ and total current from this supply is $\mathrm{I}_{1}+\mathrm{I}_{2}=4 \mathrm{~mA}$.
- The FETs have the same $\mathrm{k}_{\mathrm{n}}{ }^{\prime} \mathrm{W} / \mathrm{L}$ value.
a) [1 pt] What is the threshold voltage Vt for each FET?
b) [2 pts] Assuming M2 is in saturation mode, calculate the value of $\mathrm{k}_{\mathrm{n}}{ }^{\text {' }} \mathrm{W} / \mathrm{L}$ for these FETs.
c) [2 pts] Find the value of $\mathrm{R}_{\mathrm{S} 2}$ and a condition on $\mathrm{R}_{\mathrm{D} 2}$ that places M 2 in saturation mode.
d) [2 pts] Draw the small-signal model for this circuit (do not evaluate small-signal parameters).
e) [2 pts] Find an expression for the gain $v_{o} / v_{s}$ (do not calculate).
f) $[1 \mathrm{pt}]$ Find an expression for the input resistance, $\mathrm{R}_{\mathrm{in}}$.


## Question \#5 (13 pts)

Consider the multistage BiCMOS circuit shown here. For this circuit, you must decide whether or not to include (where appropriate) the Early Effect and CLM. You must include the Body Effect.

a) [2 pts] Assuming the BJT's have $\beta=99,\left|\mathrm{~V}_{\mathrm{BE}}\right|=0.7 \mathrm{~V}$ and $\mathrm{I}=2 \mathrm{~mA}$, determine $\mathrm{V}_{\mathrm{X}}$, the DC voltage at node X .

For the remainder of the question, you may assume that the FETs have equal $g_{m}$ and $g_{\mathrm{mb}}$ values $\left(\mathbf{g}_{\mathrm{mM} 1}=\mathbf{g}_{\mathrm{mM} 2}=\mathbf{g}_{\mathrm{mM}}\right.$ and $\left.\mathbf{g}_{\mathrm{mbM1}}=\mathbf{g}_{\mathrm{mbM}}=\mathbf{g}_{\mathrm{mbM}}\right)$.
b) [3 pts] Draw the small-signal model for this circuit (do NOT calculate smallsignal parameters)
c) $[3 \mathrm{pts}]$ Assuming $\mathrm{v}_{\mathrm{s} 2}=0 \mathrm{~V}$, find an expression for $\mathrm{v}_{\mathrm{x}} / \mathrm{v}_{\mathrm{s} 1}$ and $\mathrm{R}_{\mathrm{in} 1}$.
d) $[3 \mathrm{pts}]$ Assuming $\mathrm{v}_{\mathrm{s} 1}=0 \mathrm{~V}$, find an expression for $\mathrm{v}_{\mathrm{x}} / \mathrm{v}_{\mathrm{s} 2}$ and $\mathrm{R}_{\mathrm{in} 2}$.
e) $[2 \mathrm{pts}]$ Find an expression for $\mathrm{v}_{\mathrm{y}} / \mathrm{v}_{\mathrm{x}}$.

## Question \#6 (12 pts)

A FET differential-pair and two common-drain amplifiers are biased using currentmirrors as shown below. The diff-pair output, $\mathrm{V}_{\mathrm{o} 1}=\mathrm{V}_{\mathrm{D} 1}-\mathrm{V}_{\mathrm{D} 2}$, is amplified by the common-drain amplifier stage to produce a final differential output, $\mathrm{V}_{\mathrm{o} 2}=\mathrm{V}_{\mathrm{S} 1}-\mathrm{V}_{\mathrm{S} 2}$. All FETs in the diff-pair and common-drain stages are identical, but the ones in the current mirrors have different widths W1 and W2 as shown.


Neglect the Body effect. You must include CLM output resistances for the current mirror but neglect CLM elsewhere. Assume $V_{D D}=1.8 \mathrm{~V}$ and $\mathbf{R}_{\mathrm{D} 1}=\mathbf{R}_{\mathrm{D} 2}$.
a) [3 pts.] Analysis of the current mirrors reveals $\mathrm{V}_{\mathrm{im}}=1.2 \mathrm{~V}, \mathrm{R}_{\mathrm{im}}=12 \mathrm{k} \Omega, \mathrm{W} 1=1.2 \mu \mathrm{~m}$, and $\mathrm{W} 2=3 \mu \mathrm{~m} . \lambda=0.02 \mathrm{~V}^{-1}$ and L is identical for all devices. Estimate the output currents, $\mathrm{I}_{\mathrm{o} i}$, and output resistances, $\mathrm{R}_{\mathrm{oi}}$, at the outputs of the current mirror stages. (Note: all 3 stages have the same $\mathrm{I}_{\mathrm{oi}}$ and $\mathrm{R}_{\mathrm{oi}}$ ).
b) [1 pt.] What is the differential small-signal input resistance, Rin, of the FET diffpair?

Note: For the remaining parts, you may use given circuit parameters (i.e. $\mathbf{R}_{\mathrm{D} 1}$ ) and FET small-signal parameters. DO NOT use any numeric values.
c) [4 pts.] Find expressions for the differential output resistances, $\mathrm{R}_{\text {out1 }}$ and $\mathrm{R}_{\text {out2 }}$. (Hint: You may assume the $\mathrm{V}_{\mathrm{o} 1}$ terminals are grounded for $\mathrm{R}_{\mathrm{out} 2}$ ).
d) [4 pts.] Give expressions for the differential gains, $\mathrm{V}_{\mathrm{ol} 1} / \mathrm{V}_{\mathrm{in}}=\left(\mathrm{V}_{\mathrm{D} 1}-\mathrm{V}_{\mathrm{D} 2}\right) / \mathrm{V}_{\mathrm{in}}$ and $\mathrm{V}_{\mathrm{o} 2} / \mathrm{V}_{\mathrm{o} 1}=\left(\mathrm{V}_{\mathrm{S} 1}-\mathrm{V}_{\mathrm{S} 2}\right) / \mathrm{V}_{\mathrm{o} 1}$.

Diodes:
$i=I_{S} \exp \left(v / n V_{T}-1\right)$

BJTs:
$i_{C}=I_{S} \exp \left({ }_{B E} / V_{T}\right)$
$i_{B}=\frac{i_{C}}{\beta}$
$i_{E}=\frac{i_{C}}{\alpha}$
$i_{B}=(1-\alpha) i_{E}=\frac{i_{E}}{\beta+1}$
$\dot{i}_{E}=(\beta+1) \dot{i}_{B}$
$g_{m}=\frac{I_{C}}{V_{T}} \quad r_{e}=\frac{V_{T}}{I_{E}}=\alpha \frac{V_{T}}{I_{C}}=\frac{\alpha}{g_{m}}$
$r_{\pi}=\frac{V_{T}}{I_{B}}=\frac{\beta}{g_{m}} \quad r_{o}=\frac{V_{A}}{I_{C}}$
$r_{\pi}=(\beta+1) r_{e}$
$\beta=\frac{\alpha}{1-\alpha} \quad \alpha=\frac{\beta}{\beta+1} \quad \beta+1=\frac{1}{1-\alpha}$

FETs:
NMOS:
Cutoff:

$$
V_{G S}<V_{t} \quad I_{D}=0
$$

Triode: $\quad V_{G S}>V_{t} \quad I_{D}=k_{n}^{\prime} \frac{W}{L}\left[\left(V_{G S}-V_{t}\right) V_{D S}-\frac{1}{2} V_{D S}^{2}\right]$

$$
V_{D S}<V_{G S}-V_{t}
$$

$\begin{array}{cc}V_{G S}>V_{t} \\ V_{D S}>V_{G S}-V_{t}\end{array} \quad I_{D}=\frac{1}{2} k_{n}^{\prime} \frac{W}{L}\left(V_{G S}-V_{t}\right)^{2}\left(1+\lambda V_{D S}\right)$

Body effect: $\quad V_{t}=V_{t 0}+\gamma\left(\sqrt{2 \phi_{f}+V_{S B}}-\sqrt{2 \phi_{f}}\right)$

PMOS:
Cutoff:

$$
I_{D}=0
$$

Triode: $\quad V_{G S}<V_{t} \quad I_{D}=k_{p}^{\prime} \frac{W}{L}\left[\left(V_{G S}-V_{t}\right) V_{D S}-\frac{1}{2} V_{D S}^{2}\right]$

$$
V_{D S}>V_{G S}-V_{t}
$$

Saturation: $V_{G S}<V_{t} \quad I_{D}=\frac{1}{2} k_{p}^{\prime} \frac{W}{L}\left(V_{G S}-V_{t}\right)^{2}\left(1+\lambda V_{D S}\right)$

$$
V_{D S}<V_{G S}-V_{t}
$$

Body effect: $\quad\left|V_{t}\right|=\left|V_{t 0}\right|+\gamma\left(\sqrt{2 \phi_{f}+\left|V_{S B}\right|}-\sqrt{2 \phi_{f}}\right)$

## SMALL SIGNAL

$$
\begin{aligned}
& g_{m}=\frac{2 \cdot I_{D}}{V_{G S}-V_{t}} \\
& g_{m}=k_{n}^{\prime} \frac{W}{L}\left(V_{G S}-V_{t}\right)\left(1+\lambda \cdot V_{D S}\right) \\
& g_{m}=\sqrt{2 k_{n}^{\prime}} \sqrt{\frac{W}{L}} \sqrt{1+\lambda \cdot V_{D S}} \sqrt{I_{D}} \\
& r_{o}=\frac{1}{\lambda \cdot I_{D}} \\
& g_{m b}=\chi \cdot g_{m} \\
& \chi=\frac{\gamma}{2} \cdot \frac{1}{\sqrt{2 \phi_{f}+V_{S B}}}
\end{aligned}
$$

