

**McGill University**  
Faculty of Engineering  
Department of Electrical and Computer Engineering  
ECSE-330A – Introduction to Electronics

Examiner: Dr. David V. Plant; \_\_\_\_\_  
Associate Examiner: Dr. Mourad El-Gamal (signature on file)  
Date: Friday, December 19, 2003  
Time: 9:00 – 12:00  
Calculator: Faculty Standard

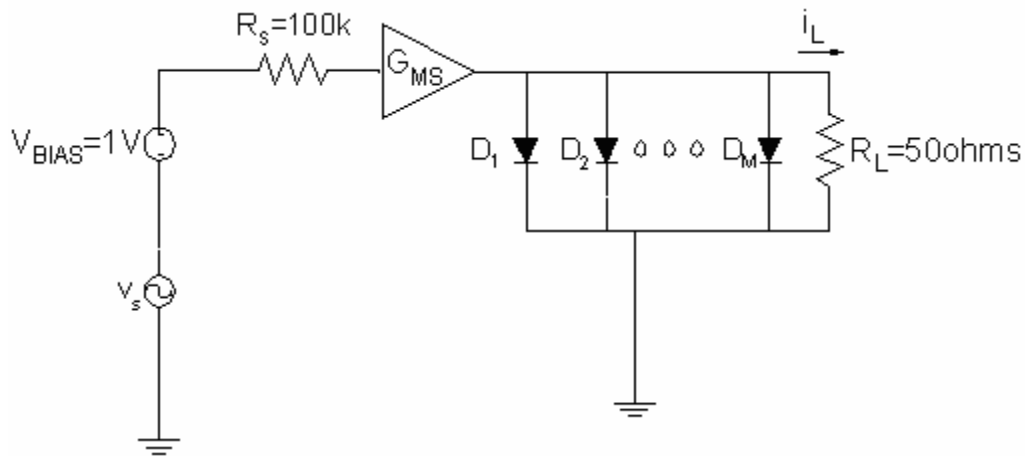
**Pertinent Information:**

- 1) This is a closed-book examination, no notes permitted. There are 3 pages of equations provided at the back of the examination.
- 2) The examination consists of 7 problems; you must answer all 7 problems.
- 3) The examination is worth 77 total points
- 4) The examination consists of 12 pages, including this page and the equations pages; please ensure you have a COMPLETE examination paper.
- 5) Only the Faculty Standard Calculator is permitted.
- 6) Questions may be completed in any order, however ensure that you clearly identify which part of which question you are attempting.

***Do NOT turn in this exam with your exam booklet***

**Question 1 (9 pts)**

The following circuit makes use of a transconductance (voltage-to-current) amplifier to create a current in  $R_L$ . The amplifier has an input resistance of  $1\text{M}\Omega$  and an output resistance of  $100\Omega$ . There are  $M$  diodes ( $n=2$ ) in parallel with the load  $R_L$ . You may use the constant voltage drop model for these diodes.



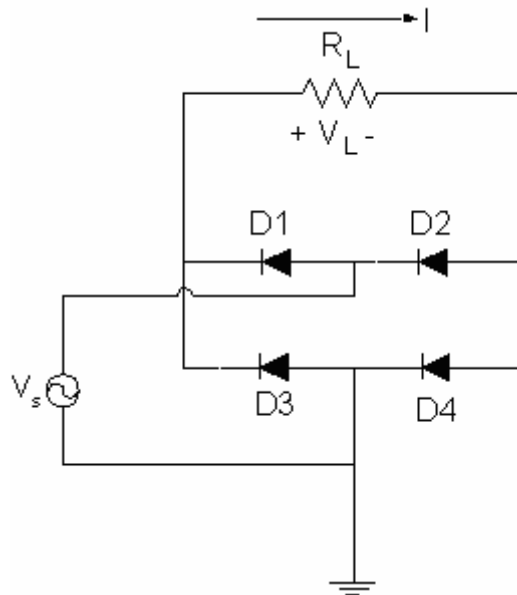
- a) [2 pts] Find a condition on the short-circuit transconductance  $G_{MS}$  so that all the diodes can conduct.

For the remaining parts of this question, let  $G_{MS} = 50\text{mA/V}$

- b) [2 pts] Determine the current in each diode as a function of  $M$ .
- c) [2 pts] Draw the small-signal model for this circuit, assuming all diodes are conducting.
- d) [3 pts] Assume there is a small input voltage ripple  $v_s = |v_s|\sin(\omega t)$  as shown. What magnitude of  $v_s$  causes a corresponding current ripple in the load of  $i_{load}(t) = 100\sin(\omega t) \mu\text{A}$ ?

**Question 2 (11 pts)**

Consider the following circuit:

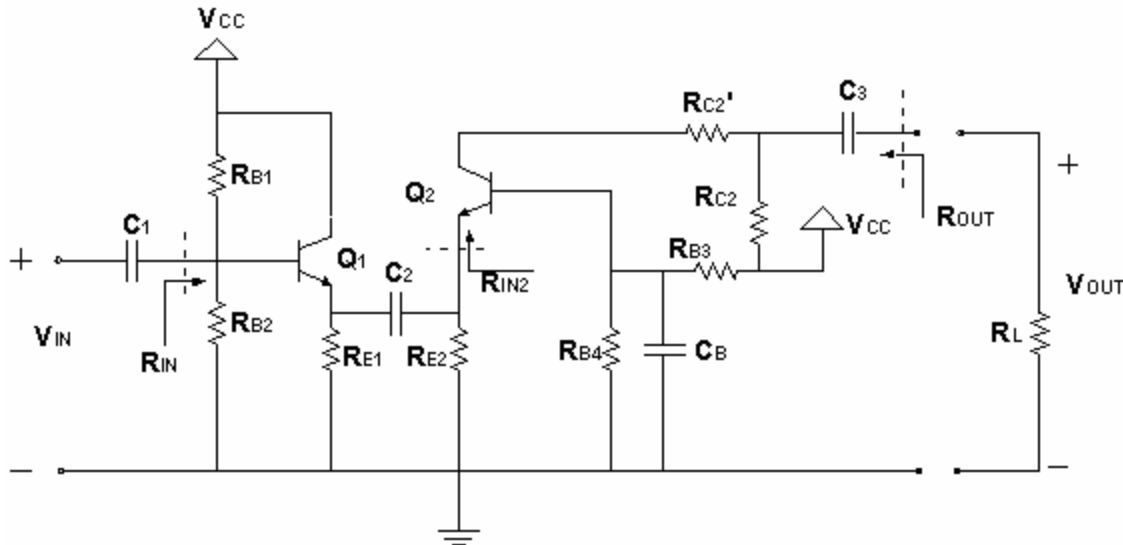


For this question, you may use the constant voltage drop model of a diode, and assume the diodes have  $n=1$ .

- [2 pts] Which diodes are on and which are off when input  $V_s$  is 5V DC?
- [2 pts] Which diodes are on and which are off when input  $V_s$  is -5V DC?
- [3 pts] Consider the AC input  $V_s = 5\sin(\omega t)$  V. Sketch a plot of this input. On the same sketch, plot the output  $V_L$  carefully, indicating any relevant voltages.
- [2 pts] Consider a small-signal input  $v_s = 10\sin(\omega t)$  mV. What is the corresponding output  $V_L$ ?
- [2 pts] What is the output if the input is  $V_s = 5 \text{ V} + 10\sin(\omega t)$  mV instead? Provide numerical answers (do not sketch).

**Question 3 (11 pts)**

The following figure shows a CC-CB two-stage amplifier. Transistors  $Q_1$  and  $Q_2$  are identical with the same  $\beta$ . All capacitors are infinite.



**For the following parts, assume:**  $V_{CC} = 12V$ ,  $R_{B1} = 120k\Omega$ ,  $R_{B2} = 40k\Omega$ ,  $R_{B3} = 120k\Omega$ ,  $R_{B4} = 20k\Omega$ ,  $R_{E1} = 3k\Omega$ ,  $R_{E2} = 1k\Omega$ ,  $R_{C2} = 10k\Omega$ ,  $R_{C2'} = 0$  and  $\beta = 49$ .

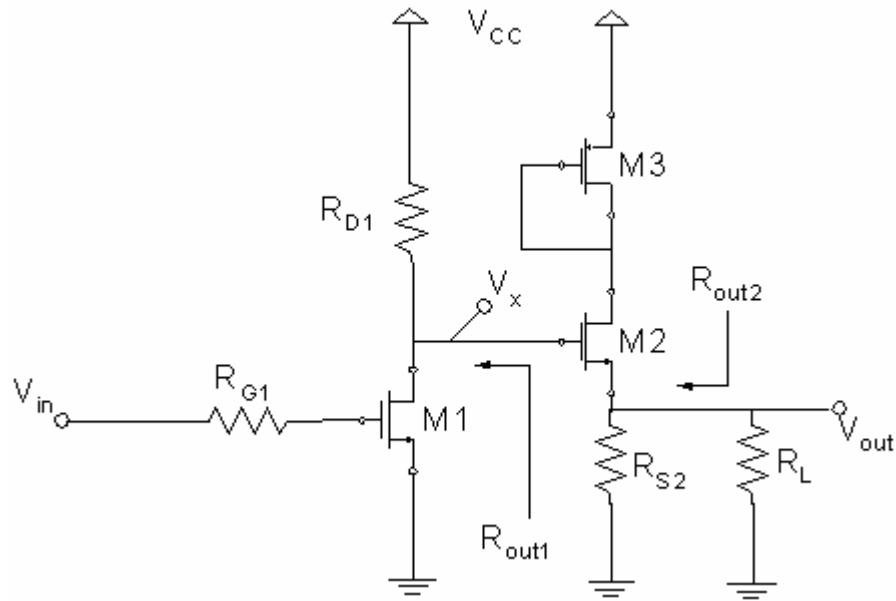
- [2 pts] Find the DC operating point of this amplifier by calculating the values of the collector current and emitter voltage for both  $Q_1$  and  $Q_2$ . You may neglect the Early effect.
- [2 pts] Draw the small signal equivalent circuit (you do not need to enter numerical values for small-signal parameters).

For parts c)-e), leave your answers in terms of small signal parameters ( $g_m$ ,  $r_p$ ,  $r_o$ ,  $r_e$  and  $\beta$ ) and the resistors shown in the schematic. You should decide where to consider the Early effect. You may use the expressions  $R_{IN}$ ,  $R_{IN2}$  and  $R_{OUT}$  in your answers.

- [2 pts] Find an expression for  $R_{IN}$
- [1 pts] Find an expression for  $R_{IN2}$
- [1 pts] Find an expression for  $R_{OUT}$
- [3 pts] Assume  $I_{C1} = I_{C2} = 0.8mA$ . If the load  $R_L$  is attached as shown, find the voltage gain  $A_v = v_{out}/v_{in}$ . You may neglect the Early effect.

#### **Question 4 (13 pts)**

Consider the following circuit. All FETs are operating in saturation. You must decide for each FET whether or not to include Channel Length Modulation and the Body Effect in your analysis.



- [2 pts] Draw the small signal equivalent circuit.
- [2 pts] Find an expression for  $V_x/V_{in}$ .
- [3 pts] Find an expression for  $V_{out}/V_x$ .
- [1 pts] Find an expression for  $V_{out}/V_{in}$ .
- [2 pts] Find an expression for  $R_{out1}$ .
- [3 pts] Find an expression for  $R_{out2}$ .

**Question 5 [10 pts]:**

Consider the following circuit. Assume  $V_{BE} = 0.7 \text{ V}$  for Q1 and Q2.

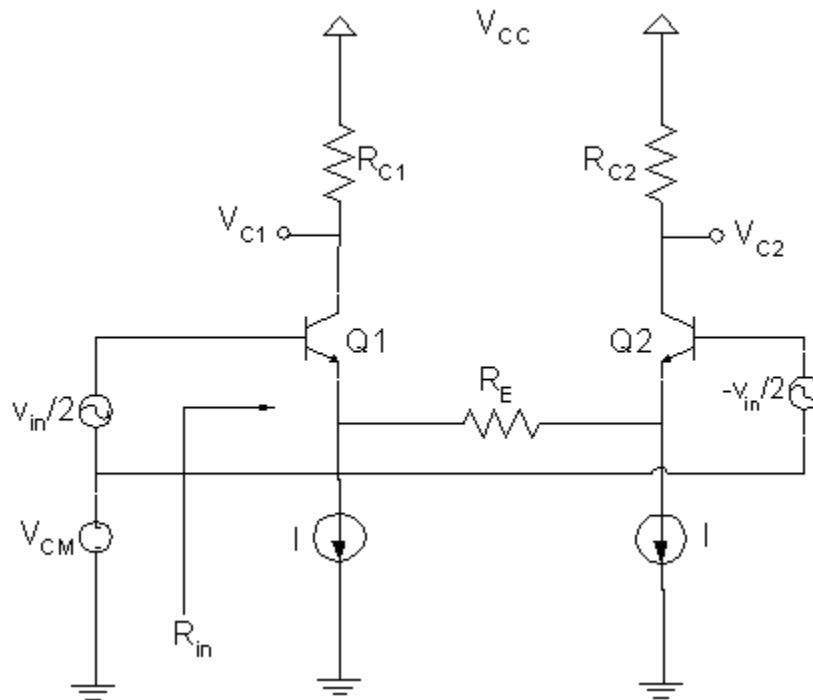
$$V_{CC} = 3.3 \text{ V}$$

$$V_{CM} = 1.8 \text{ V}$$

$$\beta = 99 \text{ for all BJTs}$$

$$V_A = 20 \text{ V for all BJTs}$$

$$I = 1 \text{ mA}$$



a) [3 pts] Calculate the value of  $R_{C1}$  and  $R_{C2}$  that simultaneously places Q1 and Q2 saturation/active boundary.

For parts (b) to (d), assume  $R_{C1} = R_{C2} = 500 \Omega$ .

b) [2 pts] Draw the small signal equivalent circuit.

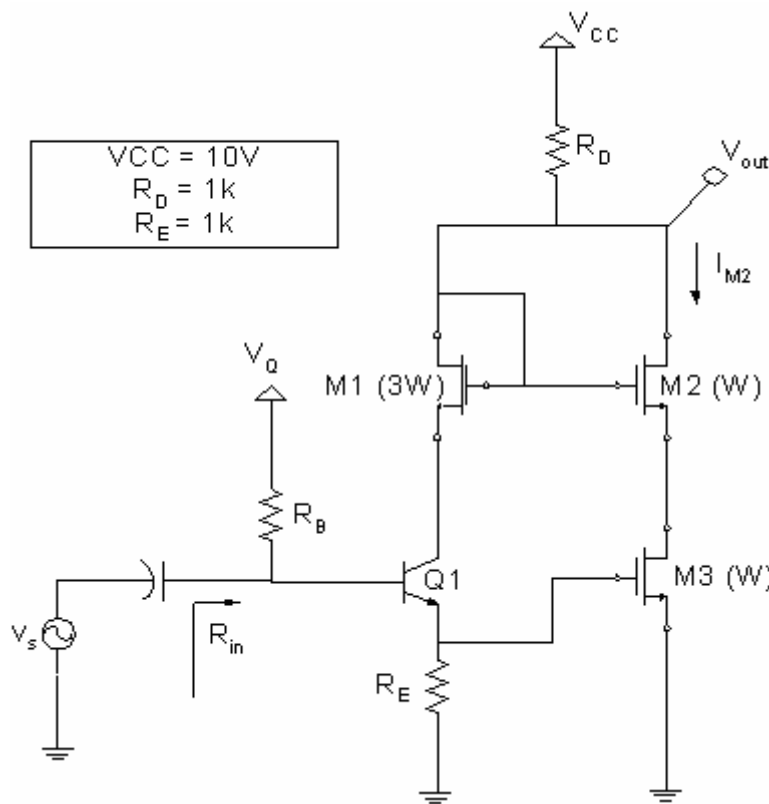
c) [2 pts] Calculate the differential voltage gain,  $v_{out}/v_{in} = (v_{c1} - v_{c2})/v_{in}$ .

d) [3 pts] Calculate the differential input resistance,  $R_{IN}$ .

**Question 6 (13 pts)**

Consider the following circuit.

- For this problem, assume that  $V_{GS-M1} = V_{GS-M2}$ .
- BJT Q1 is active and has  $\beta=99$  and  $V_{BE}=0.7$
- All FETs are in saturation and have  $V_t=1V$ .
- M2 and M3 have width  $W$ , and M1 has width  $3W$  (three times wider) as shown.
- For both DC and AC, neglect the Body Effect, the Early Effect, and Channel Length Modulation (CLM). **EXCEPTION:** You must include CLM (DC and AC) for M3 (use  $\lambda = 0.25V^{-1}$ ).



### DC Analysis

- [2 pts] Find expressions for ALL DC CURRENTS in terms of  $I_{M2}$  and ?. (If a current is 0A you do not need to write it)
- [1 pt] Express  $V_{out}$  in terms of  $I_{M2}$

- c) [2 pts] Using results from a) and b), calculate the value of  $V_{out}$  if  $I_{M2} = 1\text{mA}$ .  
(Hint: You do not need a value for  $kW/L$  to do this).

### **AC Analysis**

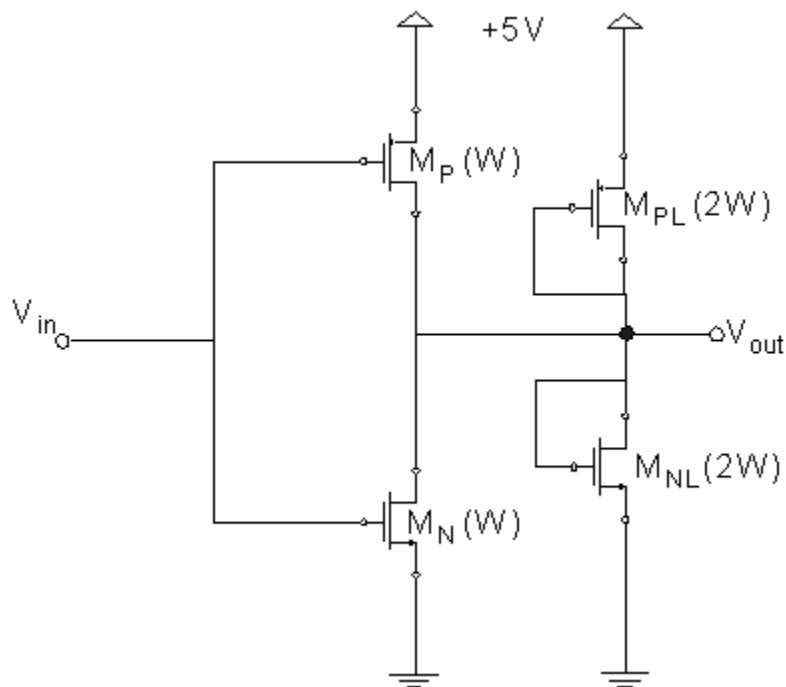
- d) [3 pts] Draw the small-signal model for this circuit, making appropriate substitutions for the diode-connected FETs M1 and M2. What is the relationship between  $g_{m_{M1}}$  and  $g_{m_{M2}}$ ?
- e) [2 pts] Find an expression for input resistance  $R_{in}$ .
- f) [3 pts] Find an expression for the small-signal gain,  $v_{out}/v_s$ , in terms of the resistances and the small-signal parameters of the FETs and BJT.

### **Question 7 (10 pts)**



Consider the following CMOS inverter circuit.

- FETs  $M_P$  and  $M_N$  have  $k_n W/L = k_p W/L = 8\text{mA/V}^2$  and  $|V_{tn}| = |V_{tp}| = 1\text{V}$ .
- The inverter is connected to a “load” formed by FETs  $M_{PL}$  and  $M_{NL}$  (of width  $2W$ ). These FETs are identical to the others, but are twice as wide.
- For this problem, ignore Channel Length Modulation and the Body Effect.



- [2 pts] Calculate  $V_{OH}$  and  $V_{OL}$  for this circuit (Hint: Identify the current that flows in each case)
- [3 pts] Sketch the voltage transfer characteristic of this device, indicating all the five regions of operation. For each region, indicate the mode of operation of each FET.
- [2 pts] Draw the small-signal model for this device, assuming all devices are in saturation.
- [3 pts] If this device received  $V_{in} = 2.5\text{ V} + 10\sin(\omega t)\text{ mV}$ , what would be the corresponding output voltage?

Diodes:

$$i = I_S \exp(v / nV_T - 1)$$

BJTs:

$$i_C = I_S \exp(v_{BE} / V_T)$$

$$i_B = \frac{i_C}{\beta}$$

$$i_E = \frac{i_C}{\alpha}$$

$$i_B = (1 - \alpha) i_E = \frac{i_E}{\beta + 1}$$

$$i_E = (\beta + 1) i_B$$

$$g_m = \frac{I_C}{V_T} \quad r_e = \frac{V_T}{I_E} = \alpha \frac{V_T}{I_C} = \frac{\alpha}{g_m}$$

$$r_\pi = \frac{V_T}{I_B} = \frac{\beta}{g_m} \quad r_o = \frac{V_A}{I_C}$$

$$r_\pi = (\beta + 1) r_e$$

$$\beta = \frac{\alpha}{1 - \alpha} \quad \alpha = \frac{\beta}{\beta + 1} \quad \beta + 1 = \frac{1}{1 - \alpha}$$

FETs:

NMOS:

Cutoff:  $V_{GS} < V_t$   $I_D = 0$

Triode:  $V_{GS} > V_t$   $I_D = k'_n \frac{W}{L} \left[ (V_{GS} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2 \right]$   
 $V_{DS} < V_{GS} - V_t$

Saturation:  $V_{GS} > V_t$   $I_D = \frac{1}{2}k'_n \frac{W}{L} (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$   
 $V_{DS} > V_{GS} - V_t$

Body effect:  $V_t = V_{t0} + \gamma \left( \sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f} \right)$

PMOS:

Cutoff:  $V_{GS} > V_t$   $I_D = 0$

Triode:  $V_{GS} < V_t$   $I_D = k'_p \frac{W}{L} \left[ (V_{GS} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2 \right]$   
 $V_{DS} > V_{GS} - V_t$

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 $V_{DS} < V_{GS} - V_t$

Body effect:  $|V_t| = |V_{t0}| + \gamma \left( \sqrt{2\phi_f + |V_{SB}|} - \sqrt{2\phi_f} \right)$

## SMALL SIGNAL

$$g_m = \frac{2 \cdot I_D}{V_{GS} - V_t}$$

$$g_m = k'_n \frac{W}{L} (V_{GS} - V_t) (1 + \lambda \cdot V_{DS})$$

$$g_m = \sqrt{2k'_n} \sqrt{\frac{W}{L}} \sqrt{1 + \lambda \cdot V_{DS}} \sqrt{I_D}$$

$$r_o = \frac{1}{\lambda \cdot I_D}$$

$$g_{mb} = \chi \cdot g_m$$

$$\chi = \frac{\gamma}{2} \cdot \frac{1}{\sqrt{2\phi_f + V_{SB}}}$$