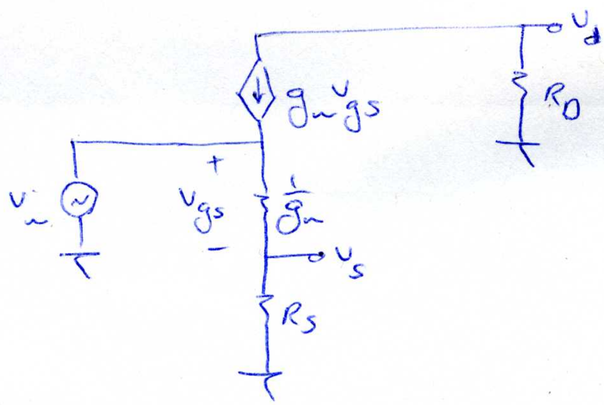
additional problems - Chapter 4 (part 2)

1

* CLM and body effect are ignored



$$v_s = \frac{R_S}{R_S + \frac{1}{g_m}} v_i \quad \text{(voltage divider)}$$

$$\Rightarrow \frac{v_s}{v_i} = \frac{R_S}{R_S + \frac{1}{g_m}}$$

$$v_{gs} = \frac{\frac{1}{g_m}}{\frac{1}{g_m} + R_S} \quad \text{(voltage divider)}$$

$$\Rightarrow v_d = -(g_m v_{gs}) R_D v_i = -g_m \frac{\frac{1}{g_m}}{\frac{1}{g_m} + R_S} R_D v_i = -\frac{R_D}{\frac{1}{g_m} + R_S} v_i$$

if $R_S >> \frac{1}{g_m}$

then $\frac{v_d}{v_i} \simeq -\frac{R_D}{R_S}$

2

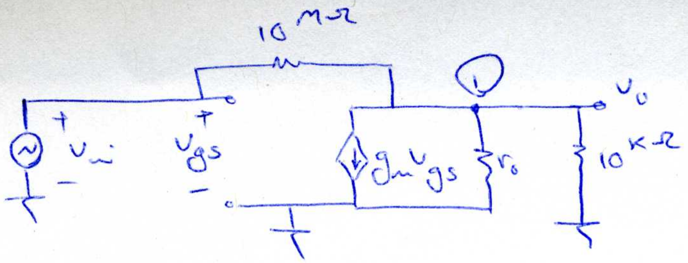
there is no DC current in the $10^{M\Omega}$ resistor ($I_G = 0$)

$\Rightarrow V_G = V_D \Rightarrow$ saturation mode ($V_{GS} > V_{DS} - V_t$)

a) $V_G = V_D = 2^V$

$r_o = \frac{1}{\lambda I_D} = 100^{k\Omega}$, $g_m = \frac{2 I_D}{V_{GS} - V_t} = 0.9091\ ^{mA}/_V$

KCL @ node ① :



$$\frac{V_o}{10^{K\Omega} \parallel r_o} + g_m V_{in} + \frac{V_o - V_{in}}{10^{M\Omega}} = 0$$

$$\Rightarrow \frac{V_o}{10^{K\Omega} \parallel 10^{M\Omega} \parallel r_o} = -\left(g_m - \frac{1}{10^{M\Omega}}\right) V_{in}$$

$$\Rightarrow \frac{V_o}{V_{in}} = \underbrace{-\ 10^{K\Omega} \parallel 10^{M\Omega} \parallel r_o \left(g_m - \frac{1}{10^{M\Omega}}\right)}_{\simeq\ -\ 10^{K\Omega} \parallel r_o\ g_m} = -8.2561\ V/V$$

b)

from part (a) :

$$500^{\mu A} = \frac{1}{2} k\frac{W}{L} (2-0.9)^2 (1 + 0.02 \overset{2^V}{\uparrow}{V_{DS}})$$

$$\Rightarrow k\frac{W}{L} = 7.9465 \times 10^{-9}$$

$$\left.\begin{array}{l} 1^{mA} = \frac{1}{2} k\frac{W}{L} (V_{GS} - 0.9)^2 (1 + 0.02 V_{DS}) \\ V_S = 0\ ,\ V_G = V_D \end{array}\right\} \Rightarrow$$

$$1^{mA} = \frac{1}{2} k\frac{W}{L} (V_D - 0.9)^2 (1 + 0.02 V_D) \Rightarrow V_D = 2.45$$

$$g_m = \frac{2 I_D}{V_{GS} - V_t} = \frac{2 \times 10^{-3}}{2.45 - 0.9} = 1.29\ mA/V$$

$$r_o = \frac{1}{\lambda I_D} = 50^{K\Omega}$$

$$\frac{V_o}{V_{in}} = -\ 10^{K\Omega} \parallel 10^{M\Omega} \parallel r_o \left(g_m - \frac{1}{10^{M\Omega}}\right) = -10.7402\ V/V$$

c) $10^{M\Omega}$ neglected : open circuit

$$\frac{V_o}{V_{in}} = -10^{K\Omega} \parallel \infty \parallel r_o \left(g_m - \frac{1}{\infty}\right) = -10^{K\Omega} \parallel r_o\ g_m = -10.75\ V/V$$

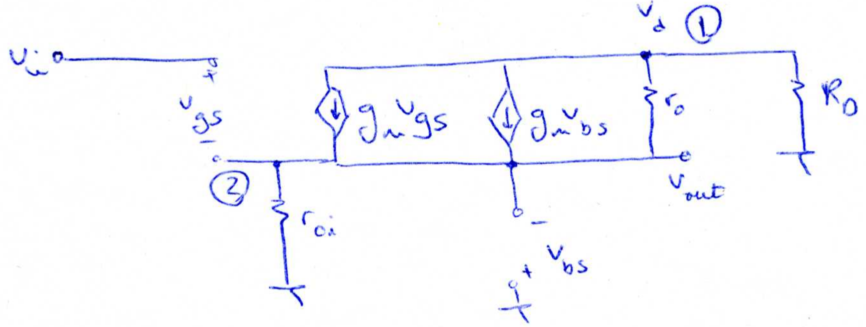
d) $V_o = -10^{k\Omega} \| 10^{M\Omega} \| r_o \left(g_m - \frac{1}{10^{M\Omega}}\right) v_{in}$

$i_{in} = \frac{v_{in} - v_o}{10^{M\Omega}}$ $\Big\}$ =>

=> $i_{in} = \frac{v_{in}}{10^{M\Omega}} \Big(1 + \underbrace{10^{k\Omega} \| 10^{M\Omega} \| r_o \left(g_m - \frac{1}{10^{M\Omega}}\right)}_{\left|\frac{v_o}{v_i}\right|}\Big) = \frac{v_{in}}{10^{M\Omega}} (1 + 10.7402)$

=> $R_{in} = \frac{v_{in}}{i_{in}} \approx 852^{k\Omega}$

#3



$v_{bs} = -v_{out}$

$v_{gs} = v_{in} - v_{out}$

$g_{mb} = X g_m$

KCL @ ① : $\frac{v_d}{R_D} = -\left(g_m v_{gs} + g_m v_{bs} + \frac{v_d - v_{out}}{r_o}\right)$

KCL @ ② : $\frac{v_{out}}{r_{oi}} = g_m v_{gs} + g_m v_{bs} + \frac{v_d - v_{out}}{r_o}$ (I)

=> $\frac{v_d}{R_D} = -\frac{v_{out}}{r_{oi}}$ => $v_d = -\frac{R_D}{r_{oi}} v_{out}$

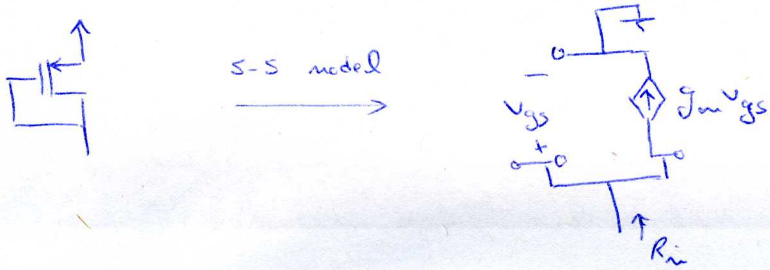
(I) => $\frac{v_{out}}{r_{oi}} = g_m (v_{in} - v_{out}) + g_{mb}(-v_{out}) - \frac{\frac{R_D}{r_{oi}} + 1}{r_o} v_{out}$

=> $v_{out}\left(\frac{1}{r_{oi}} + g_m(1 + X) + \frac{R_D}{r_{oi} r_o} + \frac{1}{r_o}\right) = g_m v_{in}$

=> $\frac{v_{out}}{v_{in}} = \frac{g_m}{g_m(1 + X) + \frac{R_D + r_o + r_{oi}}{r_{oi} r_o}}$

4

diode connected PMOS :



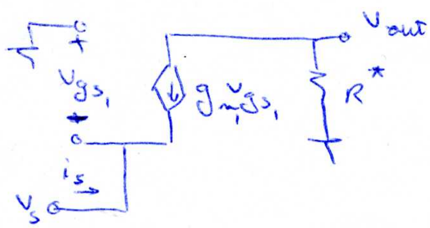
$R_{in} = \frac{V_{in}}{i_{in}}$

$V_{in} = V_{gs}$

$i_{in} = g_m V_{gs}$

$\Rightarrow R_{in} = \frac{1}{g_m}$

small-signal model :



$V_{gs_1} = -V_s$

$i_s = -g_{m_1} V_{gs_1}$

$R^* = \frac{1}{g_{m_2}}$

$V_{out} = -R^* g_{m_1} V_{gs_1} = -\frac{1}{g_{m_2}} g_{m_1} (-V_s)$

$\Rightarrow \frac{V_{out}}{V_s} = \frac{g_{m_1}}{g_{m_2}}$

$i_s = -g_{m_1} V_{gs_1} = g_{m_1} V_s$

$\frac{V_{out}}{V_s} = \frac{V_{out}}{i_s} \cdot \frac{i_s}{V_s}$

$\Rightarrow \frac{V_{out}}{i_s} = \frac{1}{g_{m_2}}$