Tutorial Feb $9^{th}/10^{th}$ (Solutions to Problems - Ch 9 FETs part 1)

1. at Triode-sat boundary: FETs have $V_{DS} = V_{GS} - V_t$

→ all FETs have current I!

→ M1: $I = \frac{1}{2}k\frac{W_1}{L_1}(V_{GS}-V_t)^2 = \frac{1}{2}k\frac{W_1}{L_1}(V_{G_1}-V_t)^2$

→ M2: $I = \frac{1}{2}k\frac{W_1}{L_1}(V_{G_2}-V_{D_1}-V_t)^2 = \frac{1}{2}k\frac{W}{L}(V_{G_2}-(V_{G_1}-V_t)-V_t)^2$

$= \frac{1}{2}k\frac{W_1}{L_1}(V_{G_2}-V_{G_1})^2$

→ M3: $I = \frac{1}{2}k\frac{W_2}{L_2}(V_{G_2}-V_t)^2$

→ all these currents are equal! ∴ $\overset{I_{M3}}{\frac{W_2}{L_2}(V_{G_2}-V_t)^2} = \underbrace{\overset{I_{M2}}{\frac{W_1}{L_1}(V_{G_2}-V_{G_1})^2} = \overset{I_{M1}}{\frac{W_1}{L_1}(V_{G_1}-V_t)^2}}$

$V_{G_2}-V_{G_1} = V_{G_1}-V_t$

$\boxed{V_{G_2} = 2V_{G_1}-V_t}$

$\frac{W_2}{L_2}(2V_{G_1}-V_t-V_t)^2 = \frac{W_1}{L_1}(V_{G_1}-V_t)^2$

$\boxed{4\frac{W_2}{L_2} = \frac{W_1}{L_1}}$

2. a) $V_y = V_x$ implies M2 is in sat mode

equate I_{M_1}, I_{M_2} ... you always have as many expressions/equations as you have FETs!

$\cancel{\frac{1}{2}k\frac{W}{L}}(V_{CC}-V_x-V_t)^2 = \cancel{\frac{1}{2}k\frac{W}{L}}(V_x-V_t)^2$

$(6-V_x)^2 = (V_x-2)^2$

$-8V_x+32=0 \rightarrow \boxed{V_x = 4}$ → we could have also found this using symmetry

b) now M2 is in triode

$\frac{1}{2}\cancel{k\frac{W}{L}}(6-V_x)^2 = \cancel{k\frac{W}{L}}\left[(V_x+3-V_t)V_x - \frac{1}{2}V_x^2\right]$

$V_x^2-12V_x+36 = 2\left[\frac{1}{2}V_x^2+3V_x-2V_x\right] \rightarrow -12V_x+36 = V_x$

$\boxed{V_x = \frac{36}{13} = 3}$

c) New V_{cc} ... $V_x = 3$, $V_y = 6$, $I = 2mA$

→ can't work with M1 (don't know V_{cc}!)

→ M2 easier: $I = 2mA = k\frac{w}{L}\left[(6-2)3 - \frac{1}{2}(3)^2\right]$

$$= \left[12 - \frac{9}{2}\right] \cdot k\frac{w}{L}$$

$$k\frac{w}{L} = 2m\left[12 - \frac{9}{2}\right]^{-1} = \boxed{.27\, mA/V^2}$$

3. For the 1st circuit:

$$I_1 = \frac{1}{2}k_n\frac{w}{L}[V_2 - 1]^2 = \frac{1}{2}k_n\frac{w}{L}(3 - V_2 - 1)^2$$

$$V_2^2 - 2V_2 + 1 = 4 - 4V_2 + V_2^2$$

$$2V_2 = 3$$

$$\boxed{V_2 = 1.5\,V} \rightarrow I_1 = \frac{1}{2}(20\mu)(3)(1.5-1)^2$$

$$= 20\mu\frac{3}{8} = 7.5\mu A$$

~~For the 2nd circuit~~

recall V_T negative!

We have: $I_3 = \frac{1}{2}k_p\frac{w}{L}[V_4 - 3 + 1]^2 = \frac{1}{2}k_n'\frac{w}{L}[V_4 - 1]^2$

$$k_p'\left[V_4^2 - 4V_4 + 4\right] = 2.5k_p'\left[V_4^2 - 2V_4 + 1\right]$$

$$1.5V_4^2 - V_4 - 1.5 = 0$$

$$V_4^2 - \frac{V_4}{1.5} - 1 = 0 \rightarrow \frac{\frac{2}{3} \pm \sqrt{\frac{4}{9} + 4}}{2}$$

(1.

check that $V_{GS} > V_T$ ... ok

$$I = \frac{1}{2}(20\mu)(3)\left[1.387 - 1\right]^2 = 4.5\mu A$$

4. Since $V_i = 5V$, $V_o < 5V$ ... probably in Triode ($V_{DS} \leq V_{GS} - 1$)

Let us set $V_o = 50mV$ (small V_{DS}! can use $r_{DS} \cong \frac{1}{k\frac{W}{L}(V_{GS} - V_t)}$

$$50\Omega \cong \frac{1}{100\mu\left(\frac{W}{L}\right)(5-1)} \Rightarrow \text{Solve } \boxed{\frac{W}{L} = 50}$$

$$V_o = 50mV \rightarrow \frac{V_{DD} - V_o}{R} = I = k\frac{W}{L}\left[(5-1)50m - \frac{1}{2}50m^2\right]$$

$$R = \frac{5 - 50m}{100\mu(50)\left[200m - \frac{1}{2}(2.5m)\right]} = \boxed{4.981\,k\Omega}$$

Also

Brief derivation of g_m for a FET amplifier

$$i_D = \frac{1}{2}k\frac{W}{L}(v_i - V_t)^2 \quad ; \quad v_o = V_{DD} - R_D i_D$$

$$\therefore v_o = V_{DD} - \frac{1}{2}R_D k\frac{W}{L}(v_i - V_t)^2$$

$$A_v = \left.\frac{\partial v_o}{\partial v_i}\right|_{v_i = V_{IQ}} \text{ (at some bias point Q)}$$

$$= -R_D k\frac{W}{L}(V_{IQ} - V_t) \quad = -R_D \times g_m \text{ where } g_m \equiv k\frac{W}{L}(V_{IQ} -$$