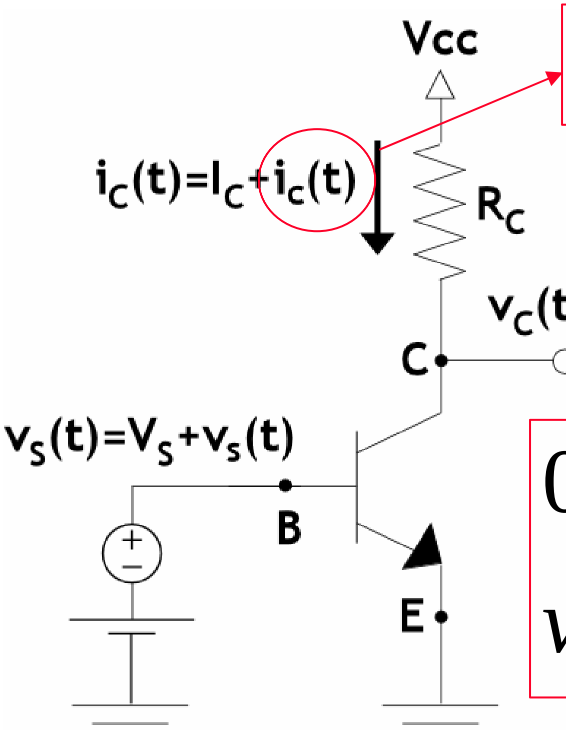
Outline of Chapter 5

- 1- Introduction to The Bipolar Junction Transistor
- 2- Active Mode Operation of BJT
- 3- DC Analysis of Active Mode BJT Circuits
- 4- BJT as an Amplifier
- 5- BJT Small Signal Models
- 6- CEA, CEA with R_E, CBA, & CCA
- 7- Integrated Circuit Amplifiers

Small Signal Model



$$i_c(t) = g_m v_{be}(t)$$

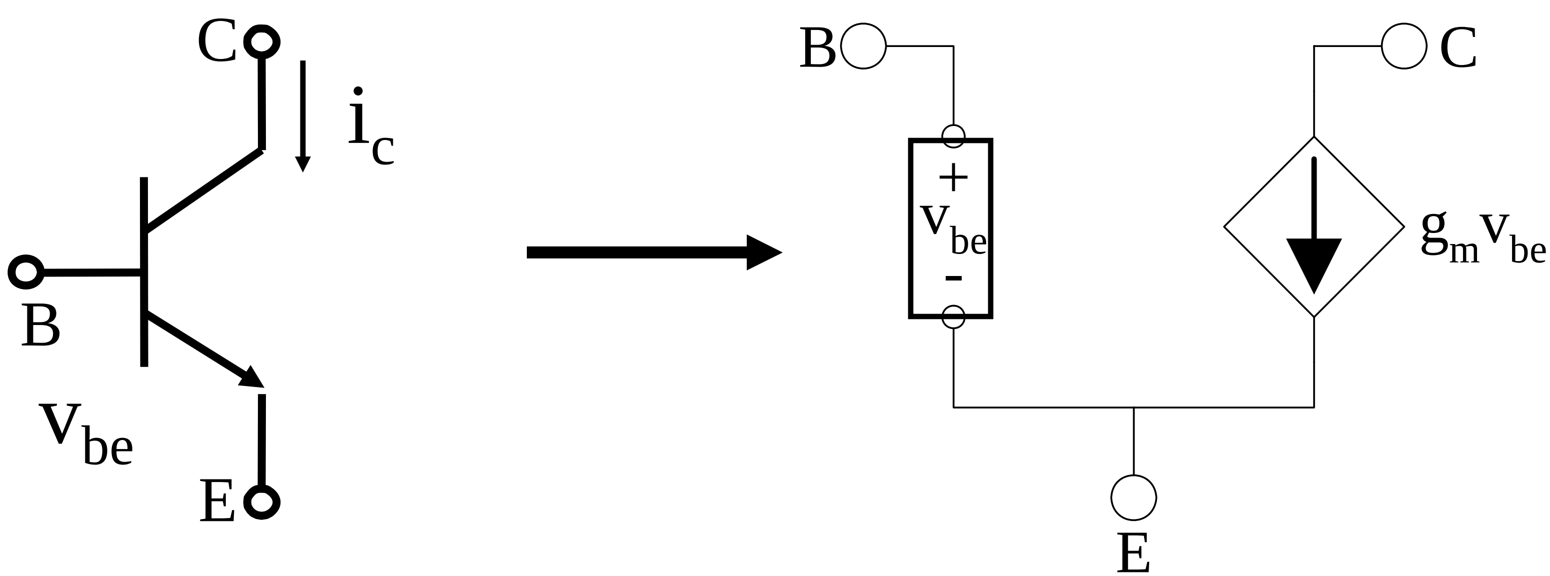
$$0 - v_c(t) = i_c R_C$$

$$v_c(t) = -g_m v_{be}(t) R_C$$

- Apply small signal at base: $v_s(t)=v_{be}(t)$
- Results in signal current, $i_c(t)$, at collector
- Can we generate a small signal model based on what we know so far

Small Signal Model: v_{be} controls i_c

- In response to signal input between base and emitter (v_{be}), signal current flows in collector (i_c)
- Can describe using hybrid-π small signal model



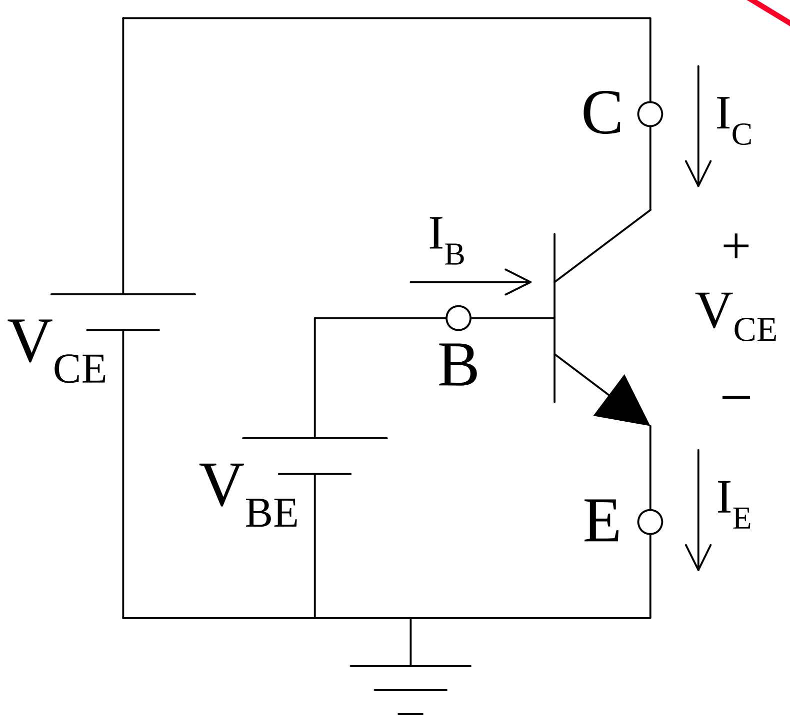
$$i_c(t) = g_m v_{be}(t)$$

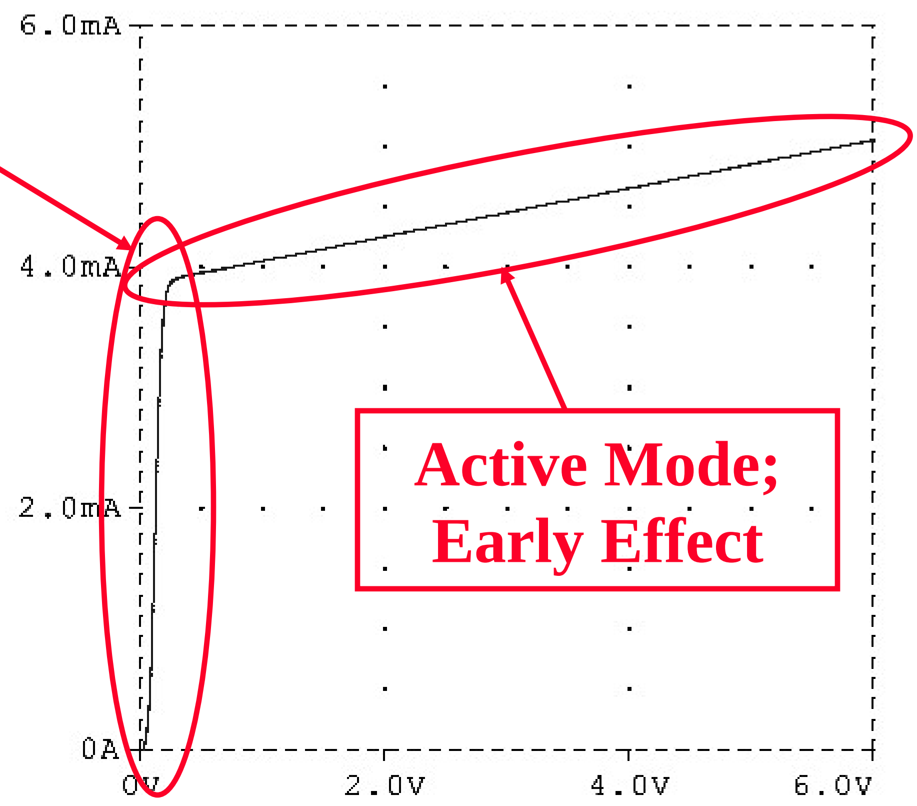
Hybrid-π small signal model

Output Resistance and Small Signal Model

- For constant V_{BE} and small V_{CE}, BJT in saturation:

Linear dependence on I_C vs. V_{CE} for constant V_{BE} defined as the Early Effect

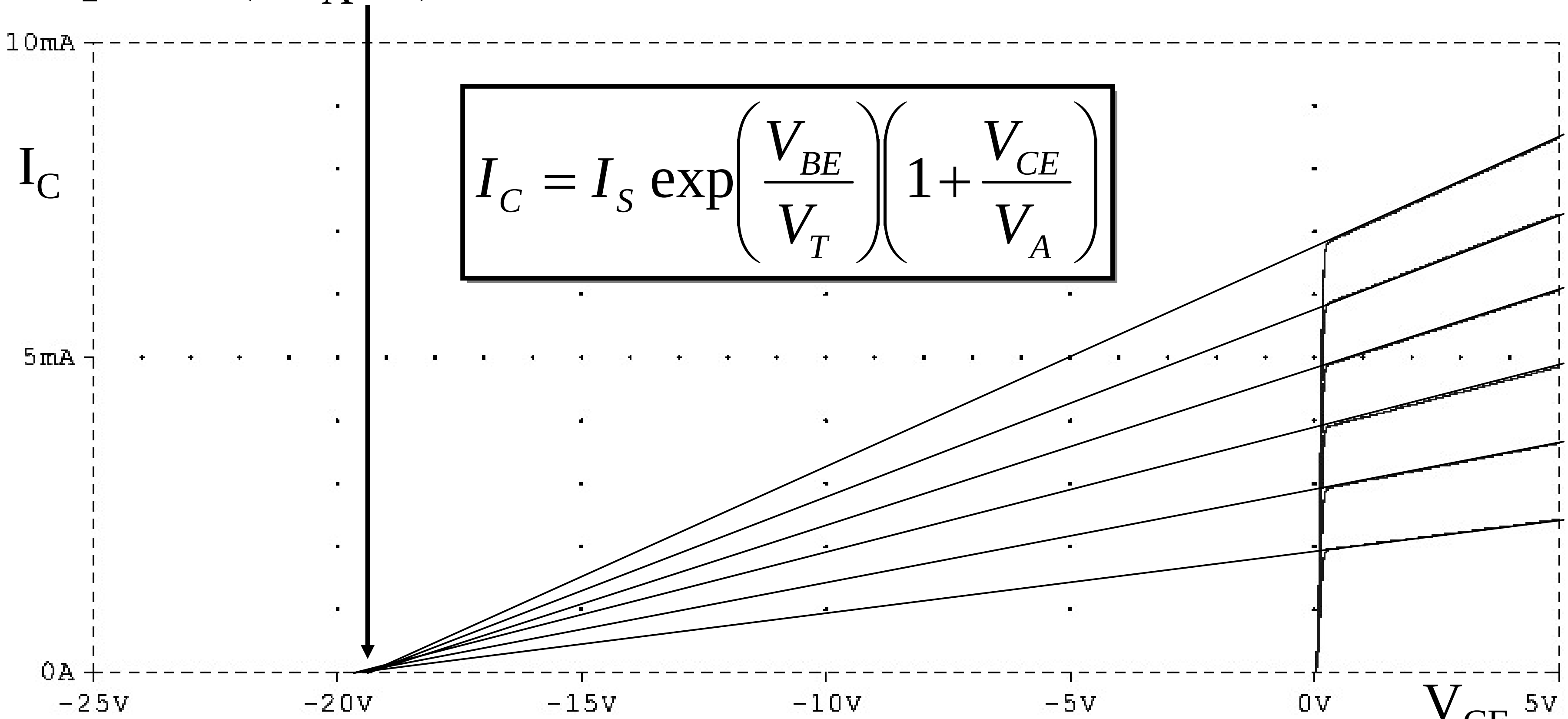




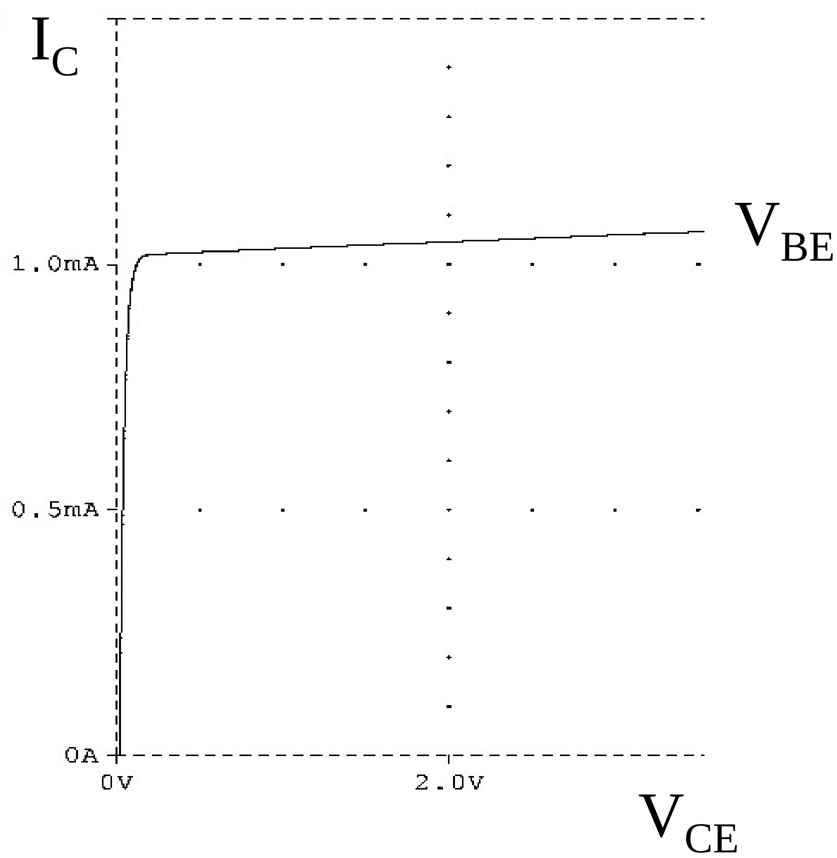
Modeling the Early Effect

- Extrapolated curves intersect at common point $(-V_A, 0)$
- V_A is the ***Early*** voltage
  - typically 50 to 100V

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE}}{V_A}\right)$$



BJT Small Signal Output Resistance

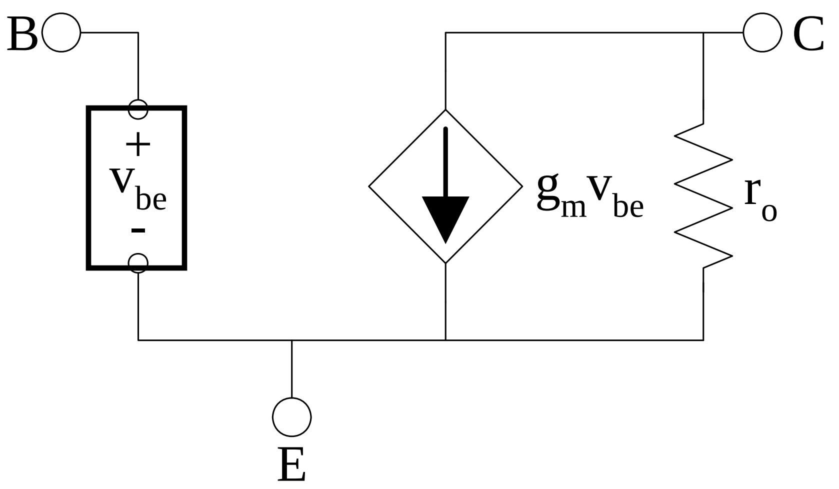


- Derivation of r_o in active mode is a small signal approximation that provides expression for BJT output resistance in the active mode:

$$r_o \equiv \left[\left. \frac{\partial I_C}{\partial V_{CE}} \right|_{v_{BE}=const.} \right]^{-1} = \frac{V_A}{I_C}$$

$$r_o = \frac{V_A}{I_C}$$

BJT Small Signal Model With Output Resistance r_o



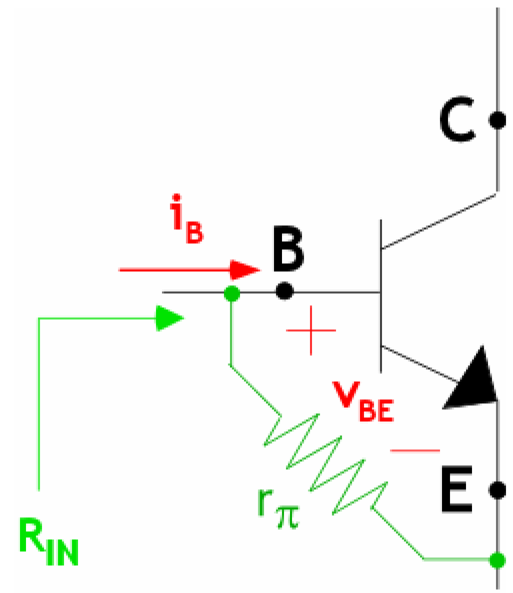
$$g_m = \frac{I_C}{V_T}$$

$$r_o = \frac{V_A}{I_C}$$

- The model is updated to include the output resistance, r_o , between C & E terminals.
- The resistance is in parallel with the current source

Small Signal: v_{be} proportional to i_b

- Since there is both a voltage and a current at the base node, there must be an equivalent resistance (V=IR)
- Small signal base current: i_b
- Small signal base emitter voltage: v_{be}
- Define small signal input resistance at base as $v_{be}/i_b = \mathbf{r}_\pi$



$$v_{be} = r_\pi \cdot i_b$$

Derive an Expression for BJT Small Signal Base Resistance – r_π

- Define expression for r_π:

$$\frac{v_{be}}{i_b} = r_\pi \equiv \left[\left. \frac{\partial I_B}{\partial V_{BE}} \right|_{OP} \right]^{-1}$$

- Make use of linearity:

$$I_B = \frac{I_C}{\beta} = \frac{I_S}{\beta} \exp\left(\frac{V_{BE}}{V_T} \right)$$

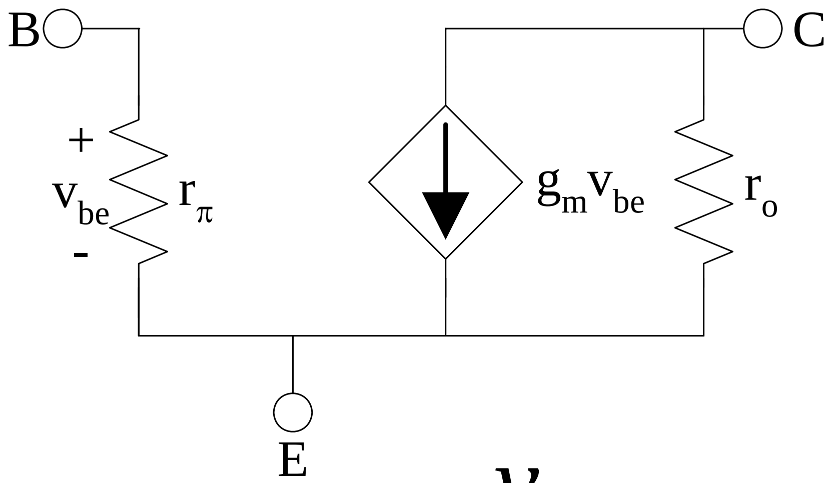
- Take derivative and simplify:

$$r_\pi = \left[\left. \frac{\partial I_B}{\partial V_{BE}} \right|_{OP} \right]^{-1} = \left[\left. \frac{1}{\beta} \frac{\partial I_C}{\partial V_{BE}} \right|_{OP} \right]^{-1} = \frac{\beta \cdot V_T}{I_C} = \frac{\beta}{g_m}$$

$$\boxed{r_\pi = \frac{\beta}{g_m} = \frac{V_T}{I_B}}$$

Hybrid-π Small Signal Models

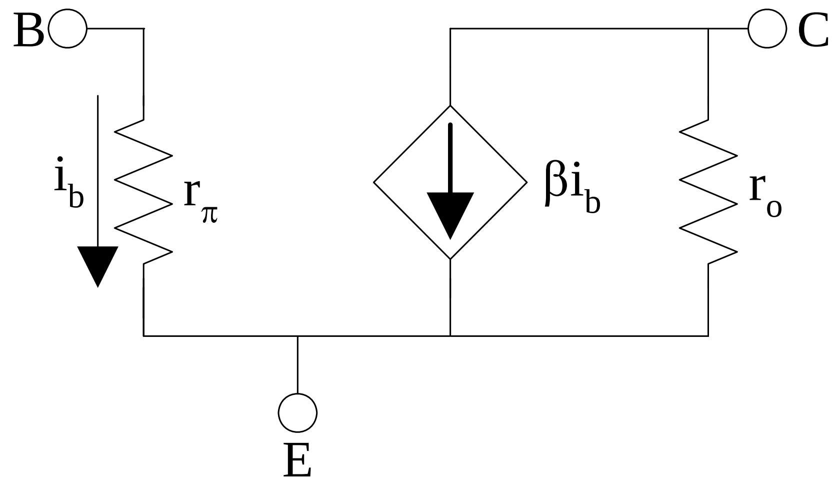
VCCS based model



$$i_b = \frac{v_{be}}{r_\pi}$$

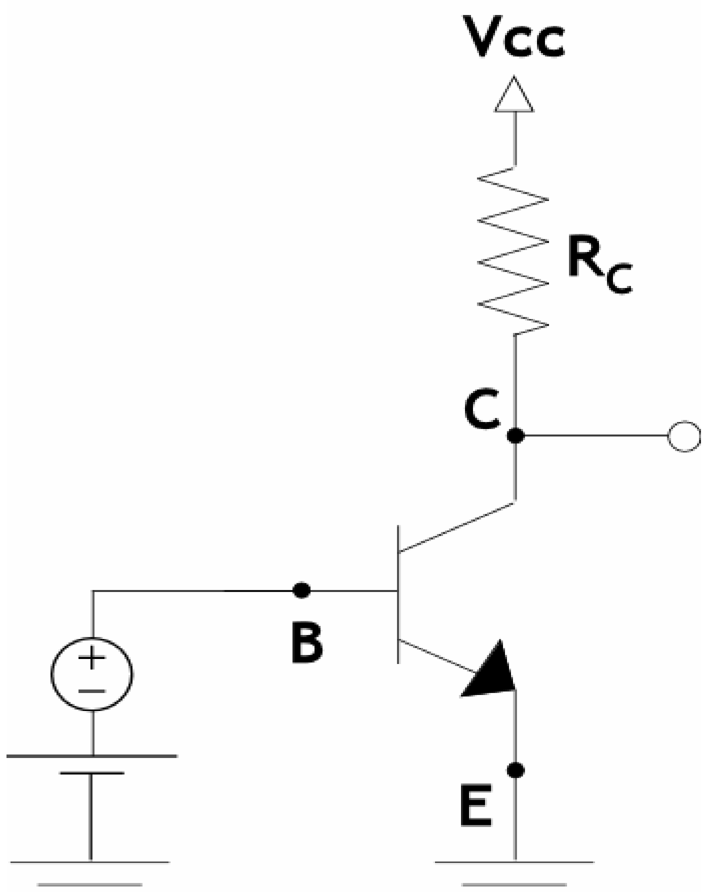
$$r_\pi = \frac{\beta}{g_m} = \frac{V_T}{I_B}$$

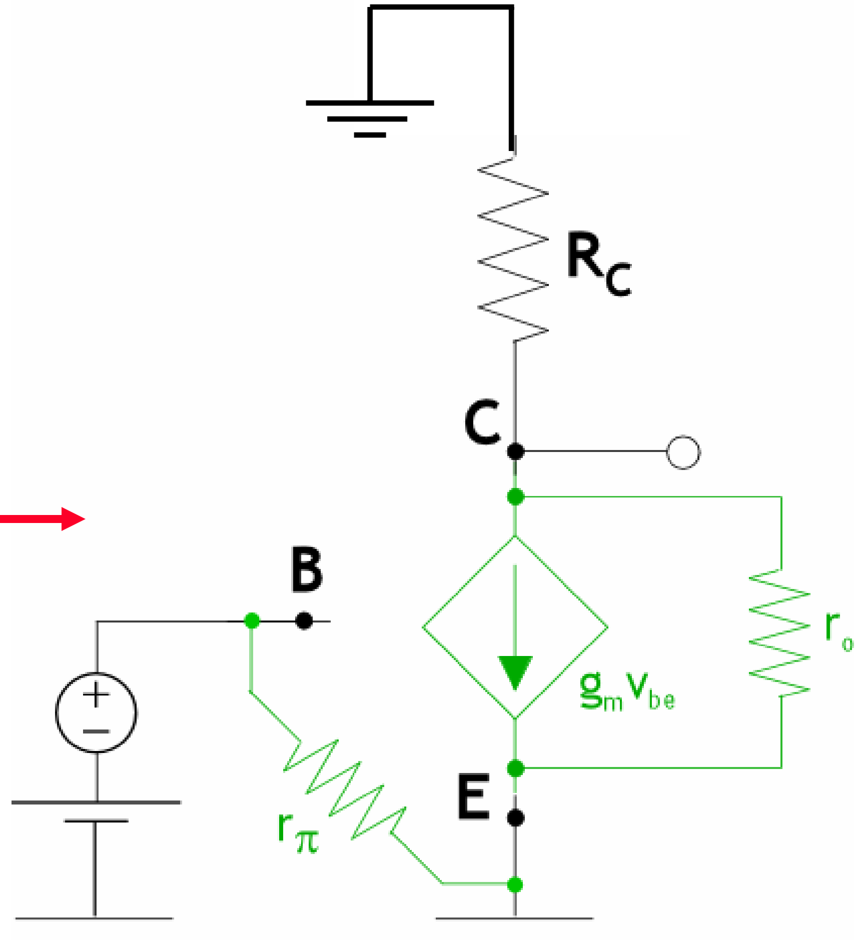
CCCS based model



$$g_m v_{be} = (g_m r_\pi) i_b = \beta i_b$$

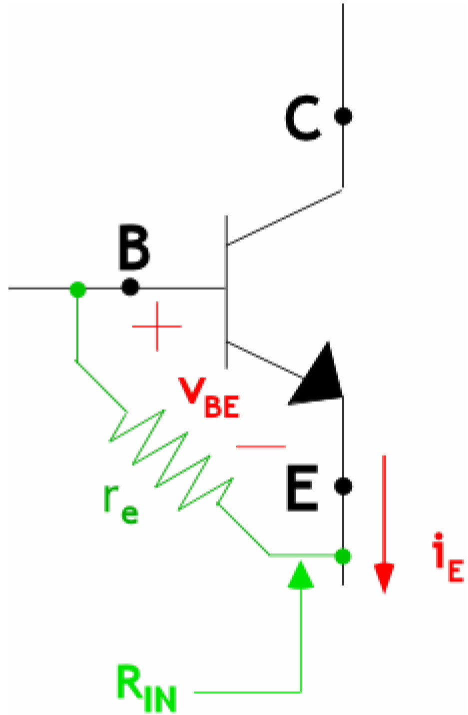
Transistor View: Inclusion of r_o and r_π





Small-Signal: v_{be} proportional to i_e

- Since there is both a voltage and a current at the emitter node, there must be an equivalent resistance (V=IR)
- Small signal emitter current: i_e
- Small signal base emitter voltage: v_{be}
- Define small signal input resistance at emitter as v_{be}/i_e= $\mathbf{r_e}$



$$v_{be} = r_e \cdot i_e$$

Derive an Expression for BJT Small Signal Emitter Resistance – r_e

- Define expression for r_e:

$$\frac{v_{be}}{i_e} = r_e \equiv \left[\left. \frac{\partial I_E}{\partial V_{BE}} \right|_{OP} \right]^{-1}$$

- Make use of linearity:

$$I_E = \frac{I_C}{\alpha} = \frac{I_S}{\alpha} \exp\left(\frac{V_{BE}}{V_T} \right)$$

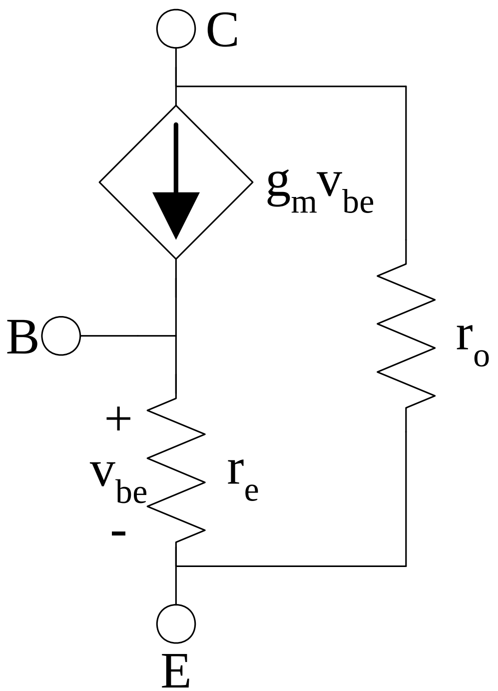
- Take derivative and simplify:

$$r_e = \left[\left. \frac{\partial I_E}{\partial V_{BE}} \right|_{OP} \right]^{-1} = \left[\left. \frac{1}{\alpha} \frac{\partial I_C}{\partial V_{BE}} \right|_{OP} \right]^{-1} = \frac{\alpha \cdot V_T}{I_C} = \frac{\alpha}{g_m}$$

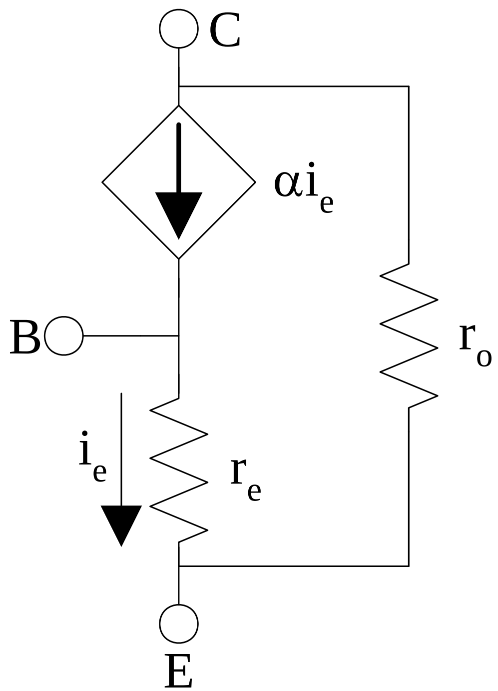
$$\boxed{r_e = \frac{\alpha}{g_m} = \frac{V_T}{I_E}}$$

T-Models

VCCS based model



CCCS based model

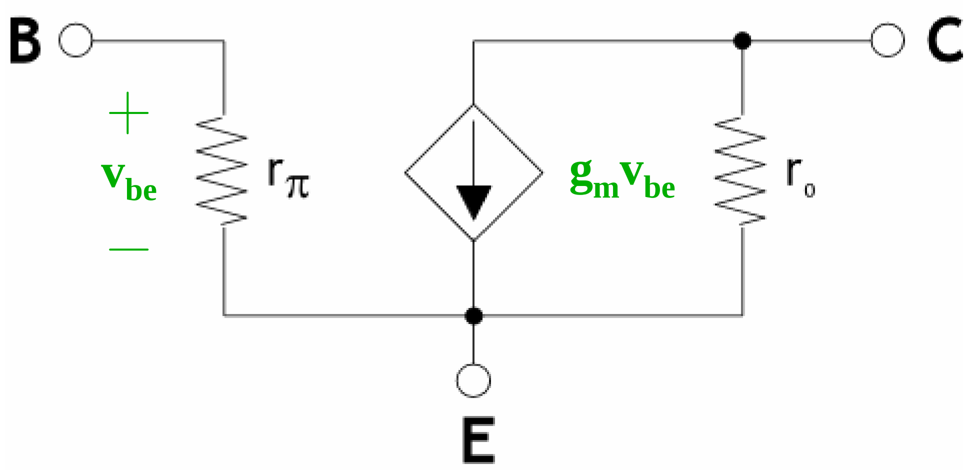


$$i_e = \frac{v_{be}}{r_e} \qquad r_e = \frac{\alpha}{g_m} = \frac{V_T}{I_E}$$

$$g_m v_{be} = (g_m r_e) i_e = \alpha i_e$$

Hybrid-π Small Signal Model – Summary

$$g_m = \frac{I_C}{V_T} \quad r_o = \frac{V_A}{I_C} \quad r_\pi = \frac{V_T}{I_B}$$



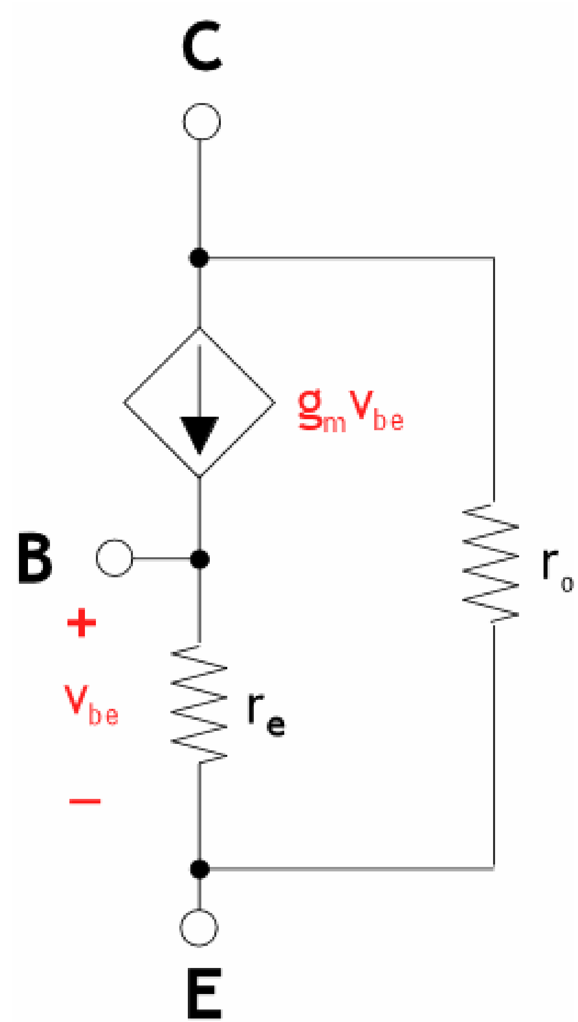
- Useful to use model when:
  - emitter terminal at signal ground
  - input applied at base

T Small Signal Model – Summary

$$g_m = \frac{I_C}{V_T} \quad r_o = \frac{V_A}{I_C} \quad r_e = \frac{V_T}{I_E}$$

- Useful to use model when:
  - input applied at base or emitter
  - emitter terminal not at signal ground
- Note:

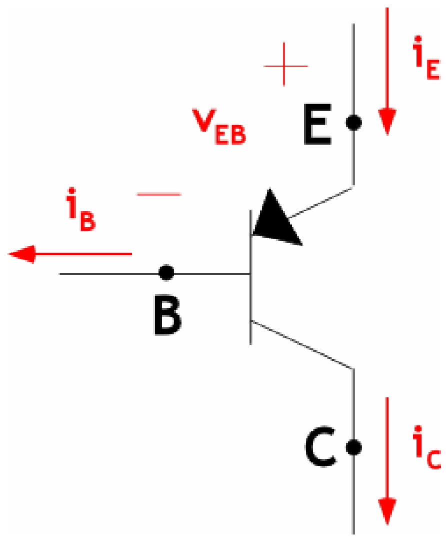
$$v_{be} = i_b r_\pi = i_e r_e$$
$$r_\pi = (i_e / i_b) r_e \;;\; i_e = (\beta + 1) i_b$$
$$r_\pi = (\beta + 1) r_e$$

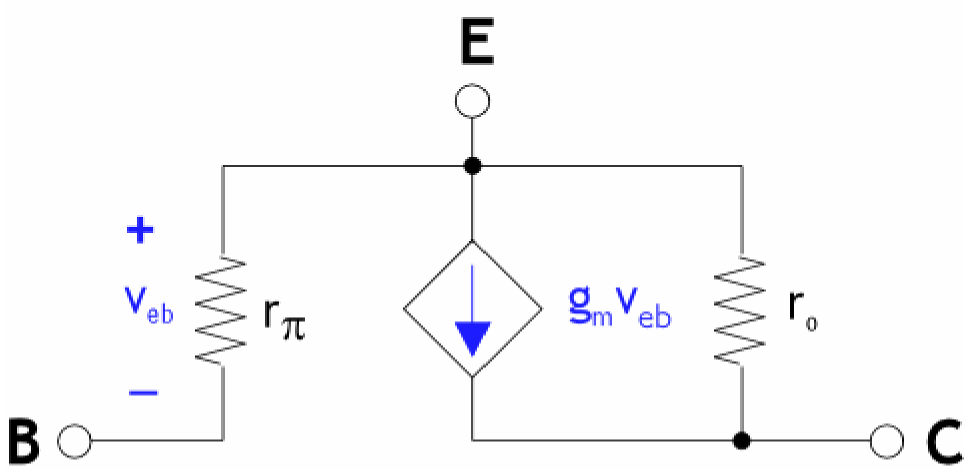


PNP Small Signal Model

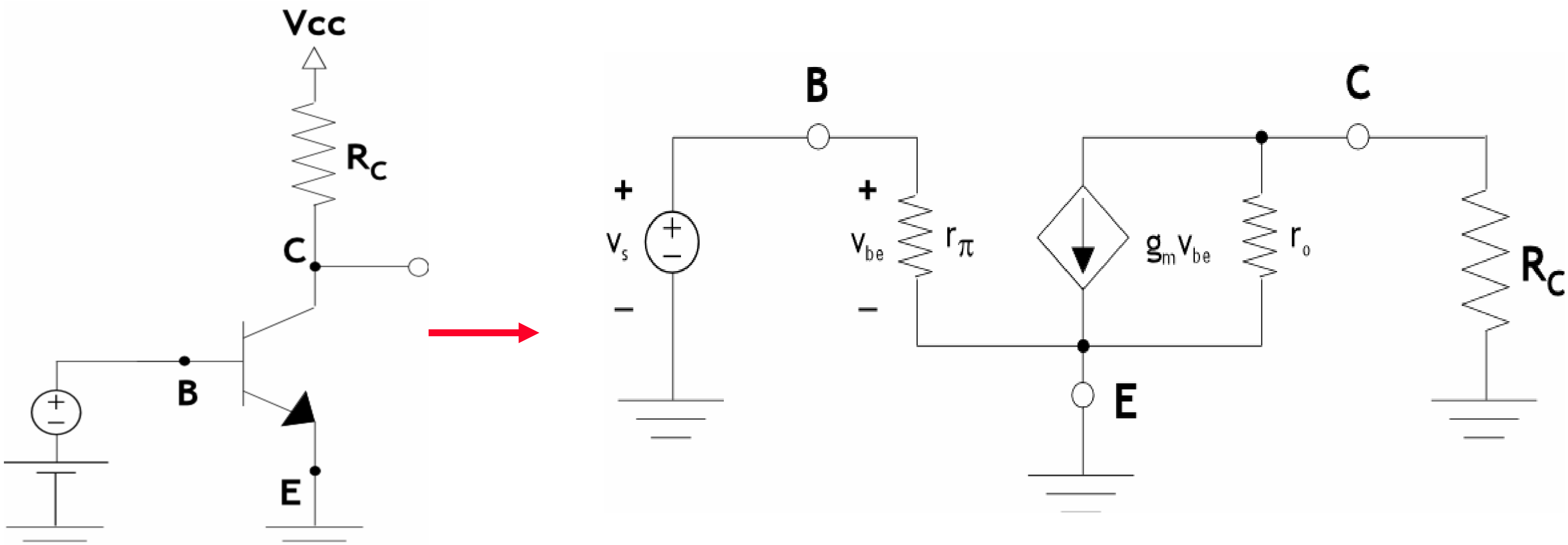
- Small-signal model for pnp's identical to npn
- Re-orient the model to reflect direction of current flow and voltage polarity
- Common mistakes:
  - Get location of B, E, and C terminals wrong
  - DC current flows one way, signal current flows other way

Note: $V_{be} = -V_{eb}$





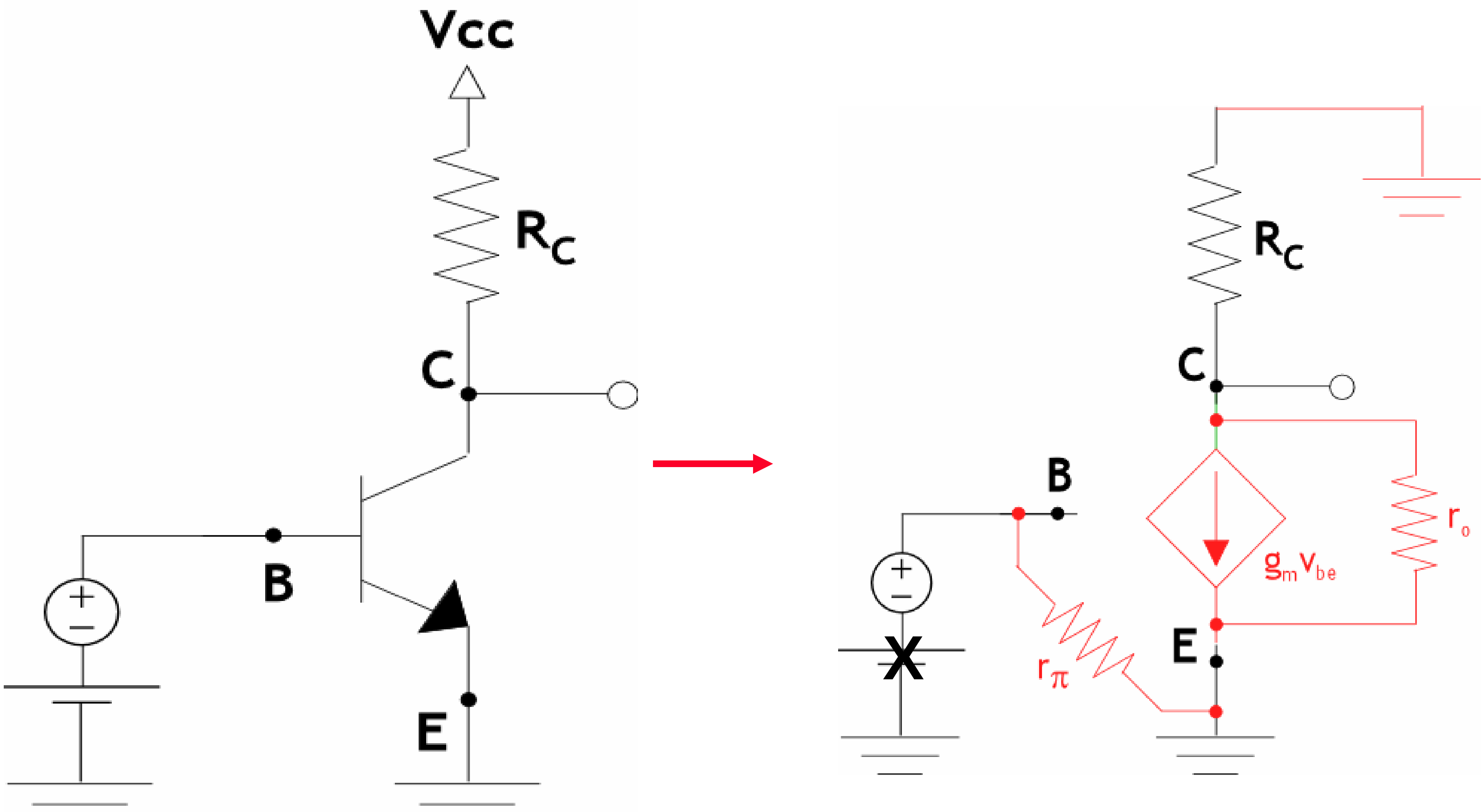
Example: Calculate Common Emitter Voltage Gain



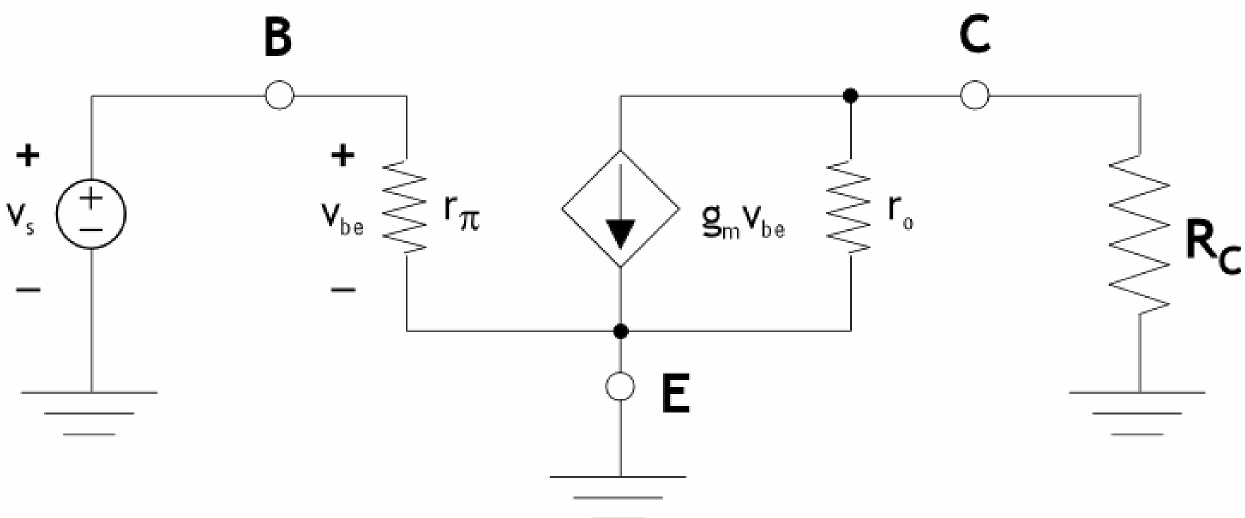
Use Hybrid-π model, and assume DC operating point is such that the following small signal parameters apply:

$$g_m = 42.4m\,{}^{A}\!/\!{}_{V} \quad r_\pi = 2.36k\Omega \quad r_o = 70.8k\Omega \quad R_C = 2k\Omega$$

Small Signal Example



Small Signal Example



$$v_s = v_{be} \qquad v_{out} = -g_m v_{be} \cdot r_o \| R_C$$

$$A_V = \frac{v_{out}}{v_s} = \frac{v_{out}}{v_{be}} = -g_m \cdot r_o \| R_C = -82.5 V/V$$

Small Signal Analysis – Comments

- Analysis procedure:
  - Solve for DC operating point
  - Calculate small-signal parameters, g_m, r_π, r_e, r_o
  - Construct small-signal equivalent circuit
  - Compute voltage and current gains, input and output resistances.
- What to do about r_o?
  - Often, including r_o complicates circuit analysis
  - Omitting r_o from small signal model results in acceptable accuracy
  - Sometimes, r_o must be included
- The often used rule of thumb: if neither end of r_o is at a signal ground, you can ignore it

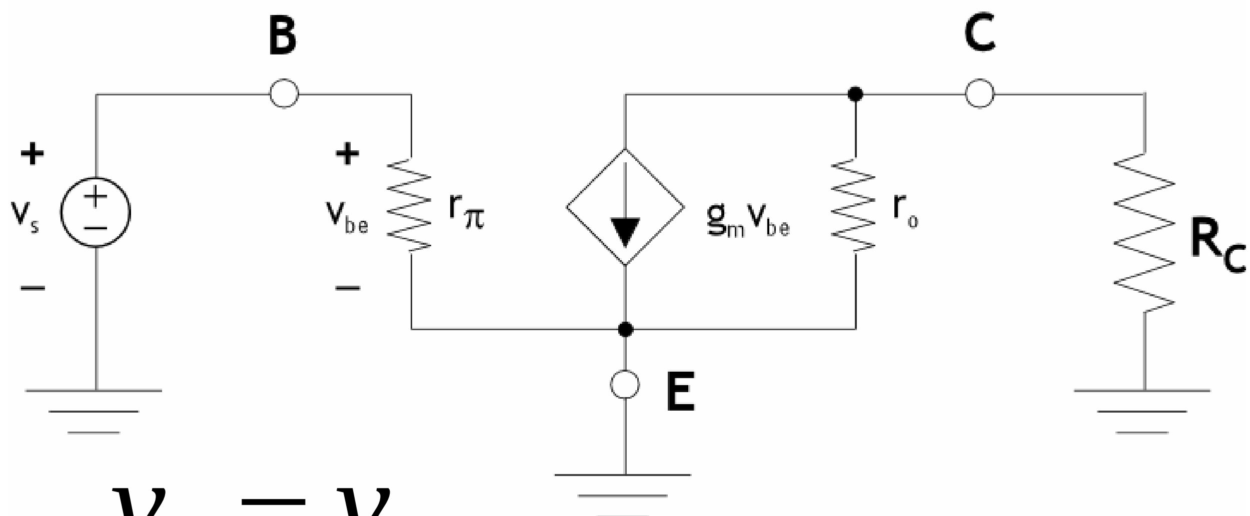
Small Signal Approximation

- Small signal analysis requires the use of "small" signals to be valid; how small is "small enough"?
- BJT based on pn junction, thus v_{be} constraint follows that for diode
- For linear approximation to be valid:

$$\frac{v_d}{n \cdot V_T} << 1 \qquad \boxed{\frac{v_{be}}{V_T} << 1}$$

- Works for v_{be} less than approximately 10mV in amplitude

What is the Limit on v_s in Order to Remain in Small Signal Approximation?



$$v_s = v_{be}$$

- For this circuit, since $v_s = v_{be}$, v_s must remain less than 10 mV in order for small signal model to be valid
- What modifications can be made to increase allowable magnitude of v_s?

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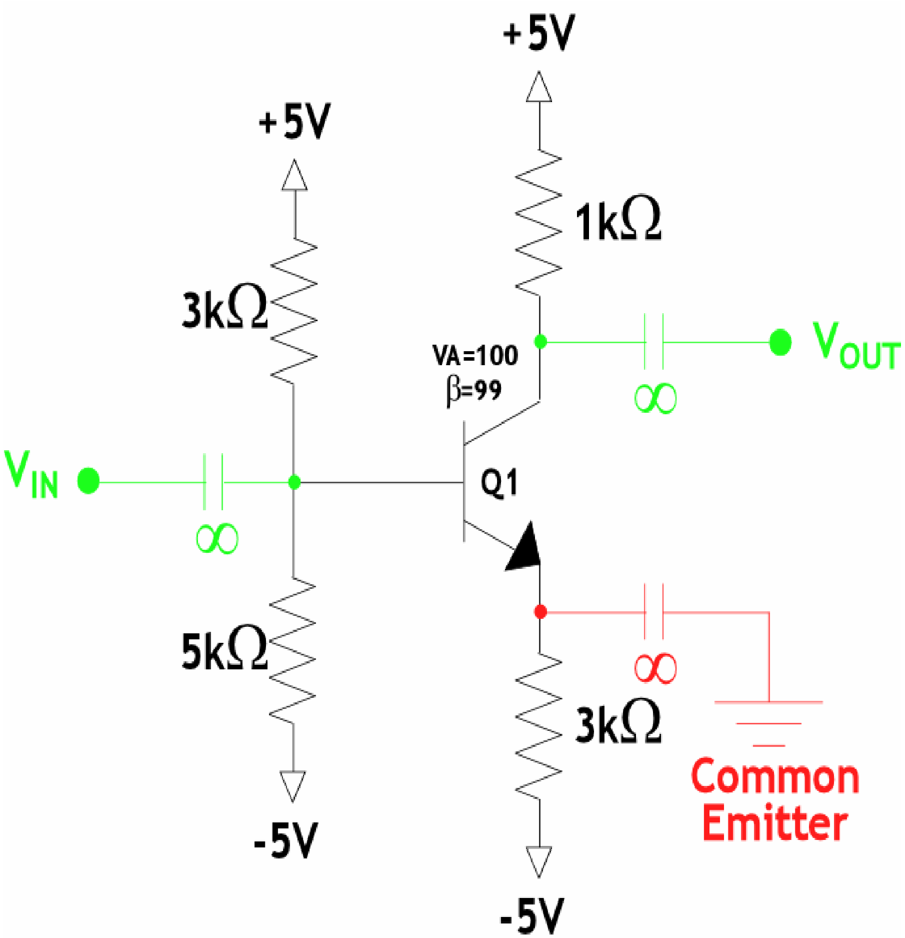
The Common Emitter Amplifier (CEA)



- Input at base; output at collector
- The emitter has a small-signal ground and is "common" to both input and output.
- By-pass (red) capacitors and coupling (green) capacitors are used to DC-bias transistor and couple signals.

$$\frac{1}{j\omega C} = 0 \quad when \quad C = \infty$$

$$\frac{1}{j\omega C} = \infty \quad when \quad \omega = 0$$

- Use Hybrid-π model with the output resistance, r_o.

CEA DC Analysis – Confirm Active

- DC Analysis
  - Thevenin:

$$V_{EQ} = (5)\left(\frac{5k}{3k+5k}\right) + (-5)\left(\frac{3k}{3k+5k}\right) = 1.25$$

$$R_{EQ} = 3000 // 5000 = 1875$$

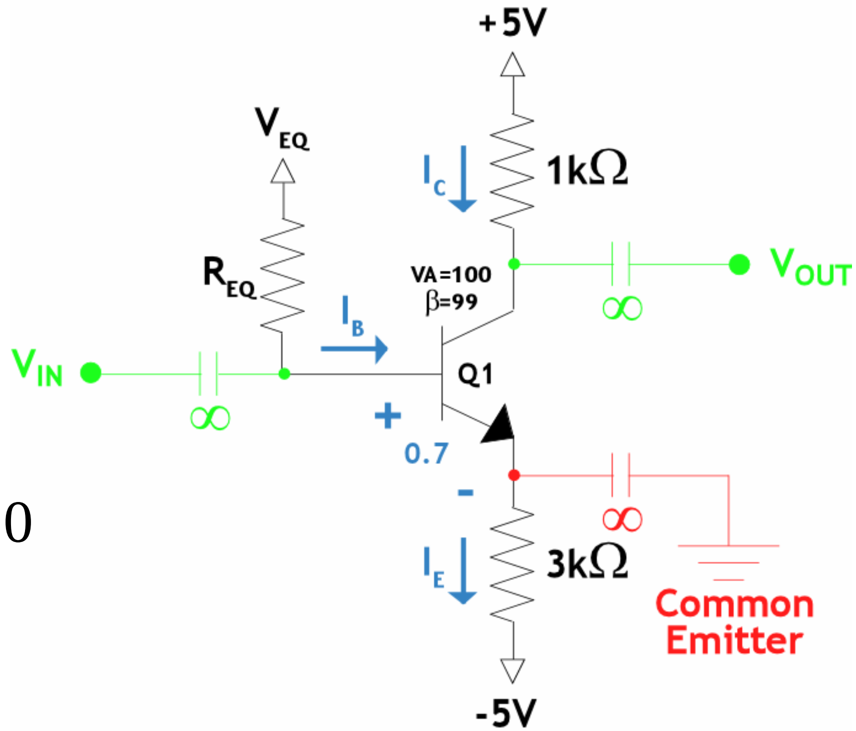
  - Analysis:

$$V_{EQ} - I_B R_{EQ} - 0.7 - I_E(3k\Omega) + 5 = 0$$

$$I_B = I_E / (\beta + 1)$$

$$I_E = \frac{V_{EQ} - 0.7 + 5}{\frac{R_{EQ}}{\beta+1} + (3k\Omega)} = 1.84mA$$

$$I_C = \frac{\beta}{\beta+1} I_E = 1.82mA$$

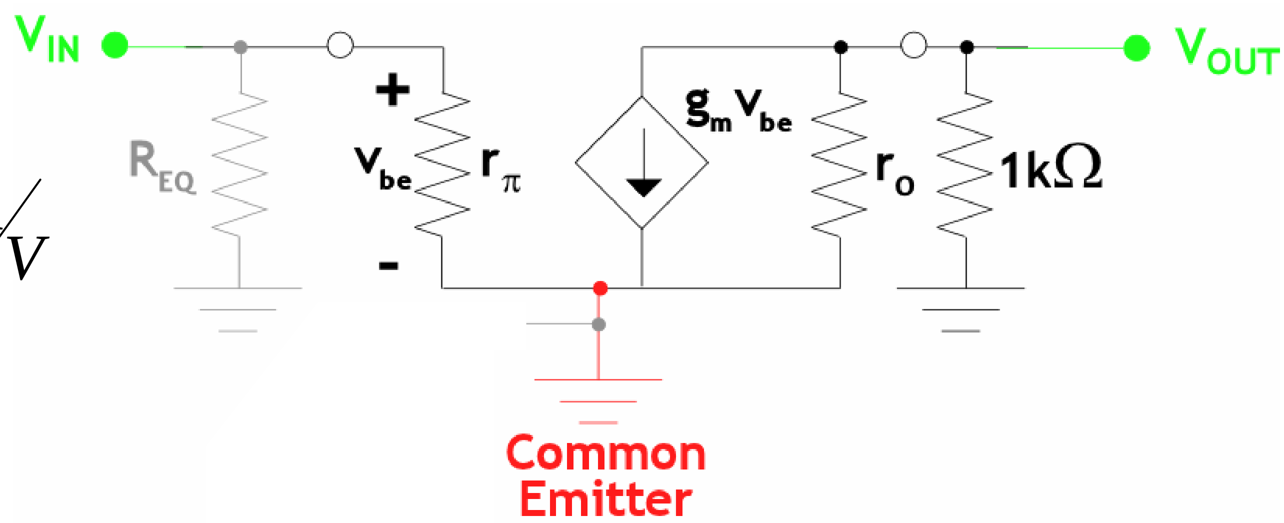


$$V_{OUT} = 5 - I_C(1k\Omega) = 3.18V$$

$$V_E = I_E(3k\Omega) - 5 = 0.515V$$

$$V_B = V_E + 0.7 = 1.215V$$

CEA Voltage Gain



$$g_m = \frac{I_C}{V_T} = 0.0728 \, A/V$$

$$r_\pi = \frac{V_T}{I_B} = 1359.4\Omega$$

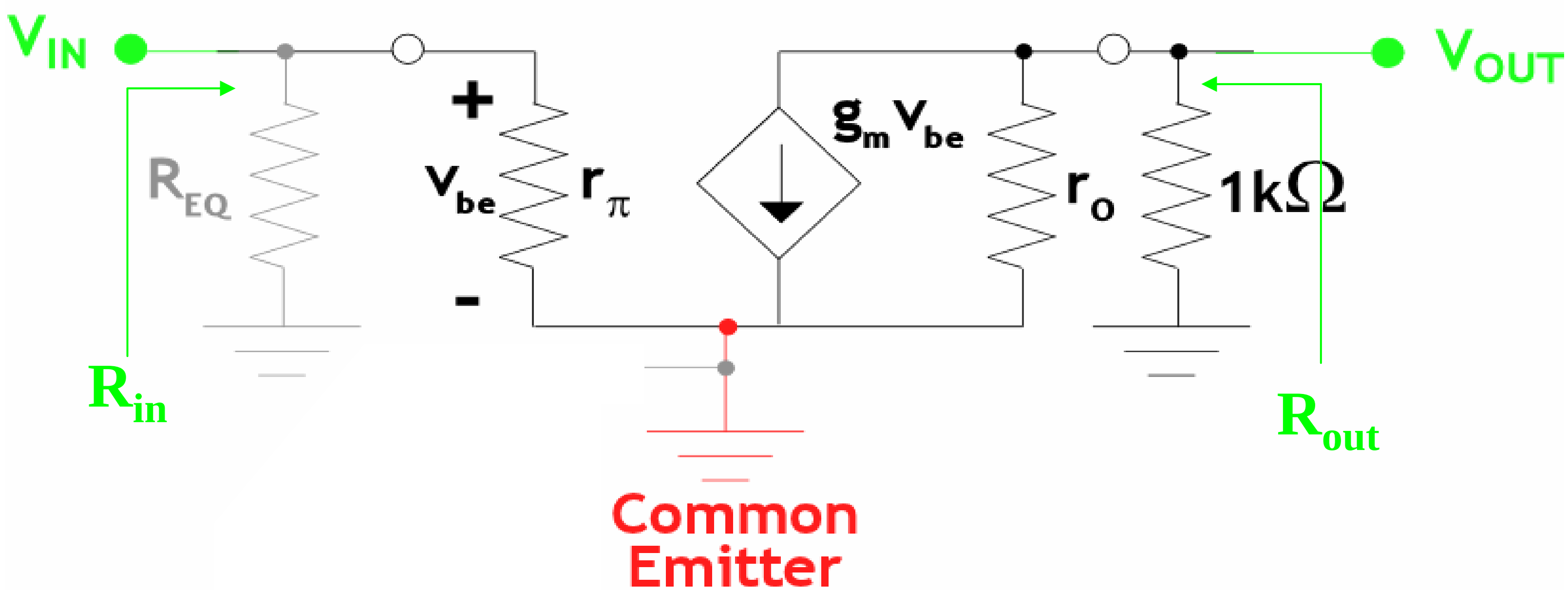
$$r_O = \frac{V_A}{I_C} = 54945\Omega$$

<u>Voltage gain analysis</u>

$$v_{be} = v_{in} \qquad v_{out} = -g_m v_{be}\left(r_o \| 1k\Omega\right)$$

$$A_{VO} = \frac{v_{out}}{v_{in}} = -g_m\left(r_o \| 1k\Omega\right)$$

CEA Input/Output Resistance



Input Resistance:

$$R_{IN} \equiv \frac{v_{in}}{i_{in}}$$

$$R_{IN} = r_{\pi} \,||\, R_{EQ} = 786\Omega$$

Output resistance: set $v_{in} = 0$; thus $v_{be} = 0$; leads to $g_m v_{be} = 0$ and thus *open circuited*; note one node of r_o tied to ground

$$R_{OUT} = r_o || 1k\Omega = 982\Omega$$

CEA Summary

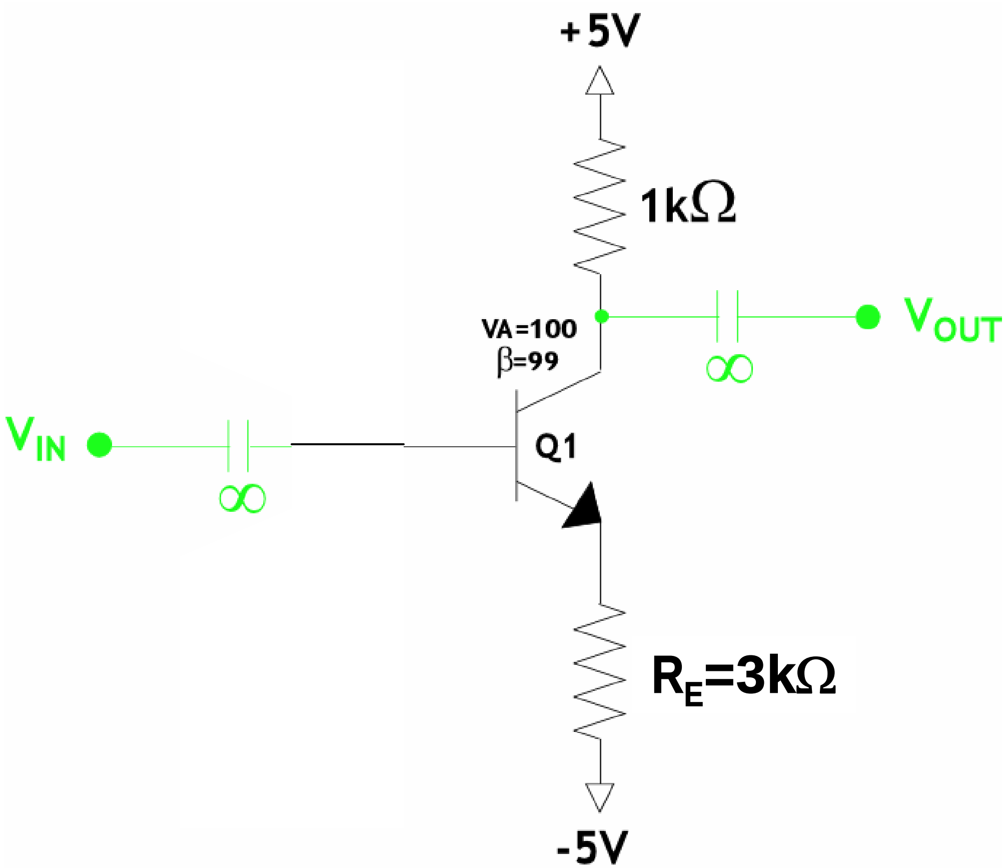
$$A_{VO} = \frac{v_{out}}{v_{in}} = -g_m\left(r_o \| R_C\right)$$

$$R_{IN} = r_\pi$$

$$R_{OUT} = r_o \| R_C$$

- Large gain parameters
- R_{IN} & R_{OUT} both moderate
- R_{IN} & R_{OUT} not ideal
  for any amplifier configuration
- In practice, not usually used by itself for gain

CEA with R_E



- There is NO small-signal ground in the emitter
- Since there is neither collector nor emitter at signal ground
  - Use T small-signal model
  - Neglect r_o

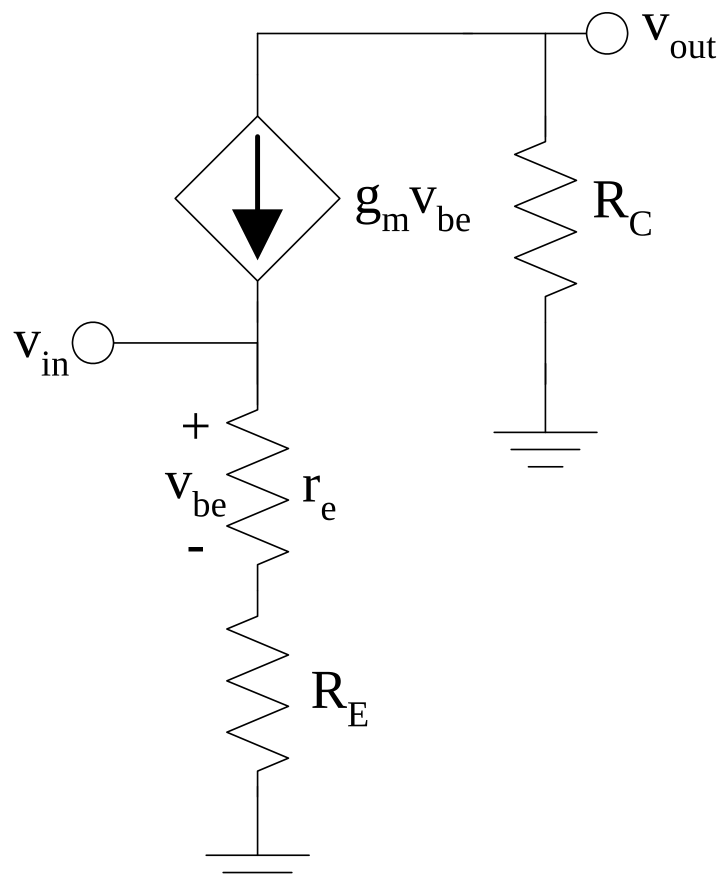
CEA with R_E - Voltage Gain Analysis

- Draw small-signal T-model equivalent circuit.
- Neglect r_O
- Voltage gain

$$v_{be} = \frac{r_e}{r_e + R_E} v_{in}$$

$$v_{out} = -g_m v_{be} R_C$$

$$A_{VO} = \frac{v_{out}}{v_{in}} = -g_m R_C \frac{r_e}{r_e + R_E}$$



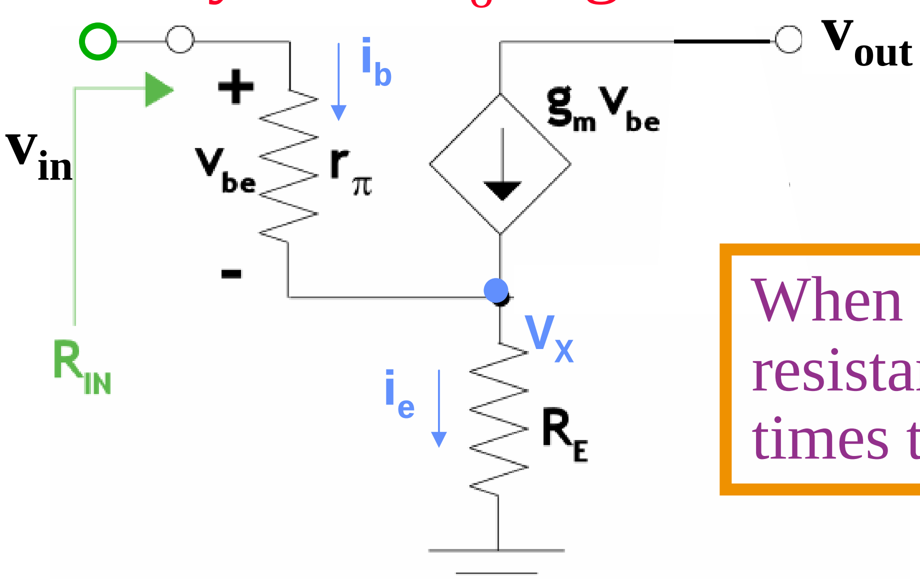
R_{IN}: the β+1 Reflection Rule

- The β+1 rule takes advantage of the relationship between i_E and i_B
- Rule generally applicable only when r_o neglected

$$R_{IN} \equiv \frac{v_{in}}{i_{in}} = \frac{v_{be} + v_x}{i_b} = \frac{i_b r_\pi + i_e R_E}{i_b}$$

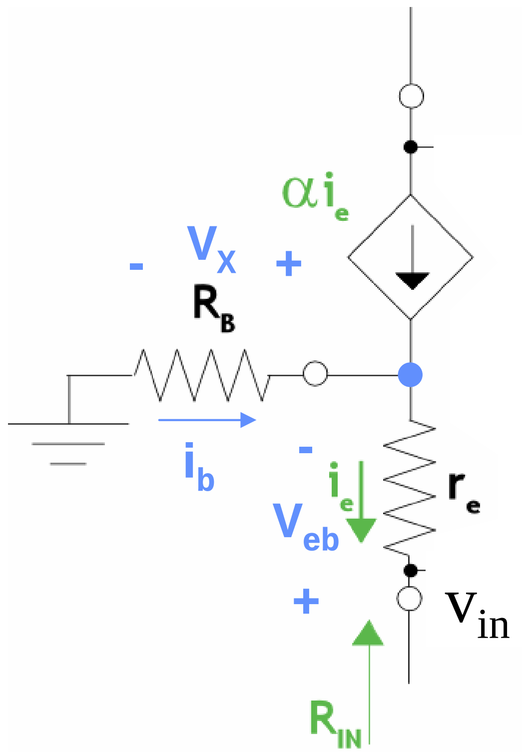
$$i_e = (\beta + 1) i_b$$

$$R_{IN} = r_\pi + (\beta + 1) R_E$$



When looking into the base, the input resistance is the base resistance plus (β+1) times total resistance in the emitter

The β+1 Reflection Rule in Reverse: Looking Into the Emitter



$$v_{in} = v_{eb} + v_x \; ; \; v_{in} - v_{eb} = v_x$$

$$R_{IN} \equiv \frac{v_{in}}{i_{in}} = \frac{v_{eb} + v_x}{-i_e} = \frac{-i_e r_e - i_b R_B}{-i_e}$$

$$i_e = (\beta + 1) i_b \quad R_{IN} = r_e + \frac{R_B}{(\beta + 1)}$$

- R_B looks (β+1) ***smaller*** from emitter perspective
- When looking into the emitter, the input resistance is the emitter resistance ***PLUS*** whatever is in the base divided by (β+1)

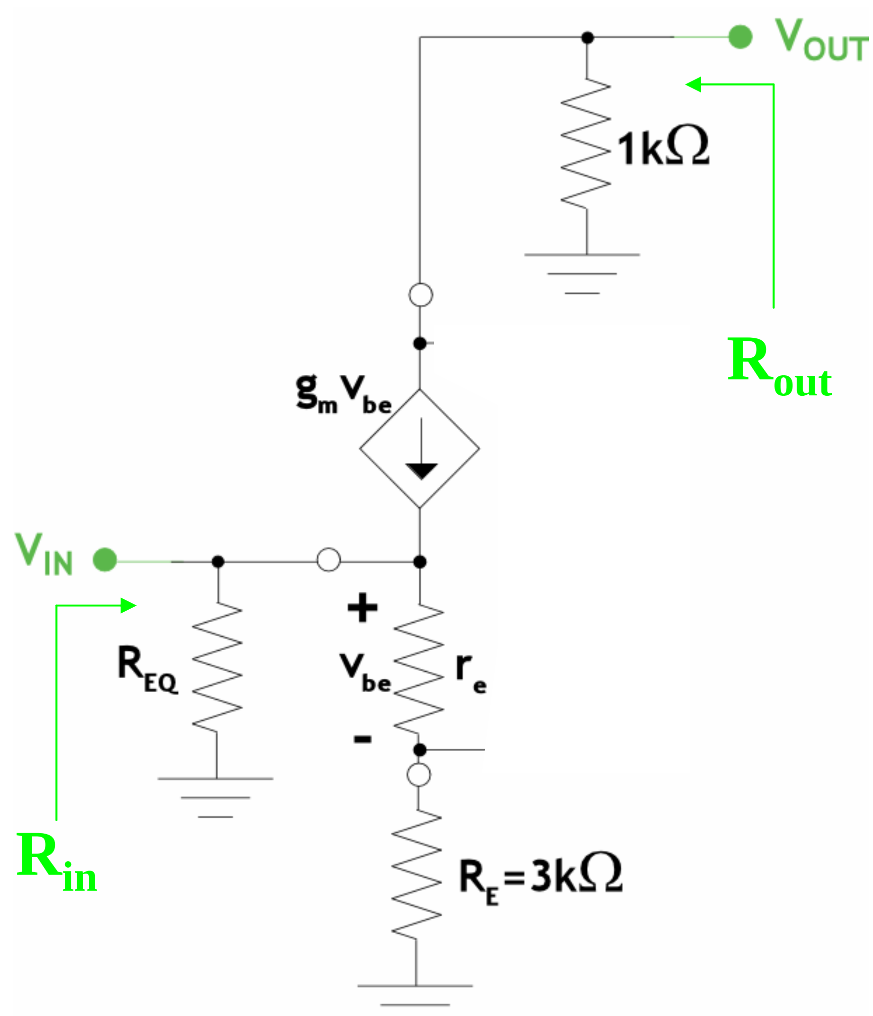
CEA with R_E– Input/Output Resistance

- Neglect r_O
- Input Resistance
- Using the β+1 reflection rule:

$$R_{IN} = R_{EQ} \,||\, [(\beta+1)(r_e + R_E)]$$

- Output resistance: set v_{in} = 0; thus v_{be} = 0; leads to $g_m v_{be}$ = 0 and thus *open circuited;* by inspection:

$$R_{OUT} = 1k\Omega$$



The CEA With R_E – Summary

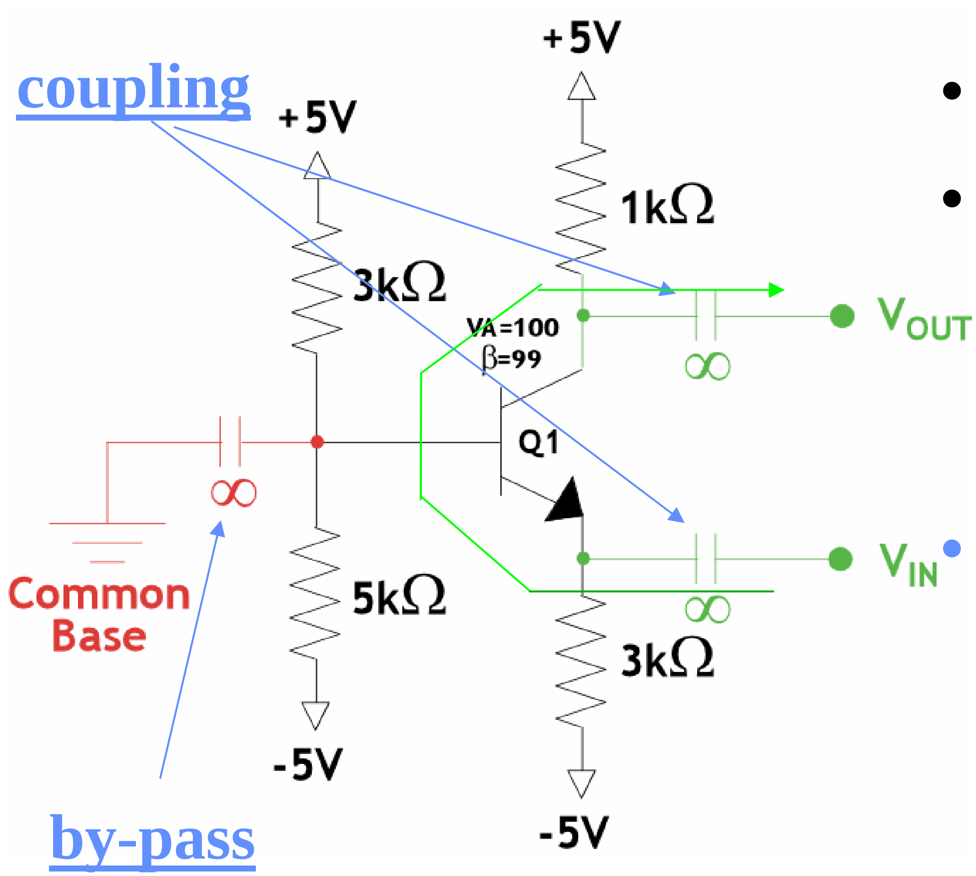
$$A_{VO} = \frac{v_{out}}{v_{in}} = -g_m R_C \frac{r_e}{r_e + R_E}$$ moderate

$$R_{IN} = (\beta + 1)(r_e + R_E)$$ large

$$R_{OUT} = R_C$$ moderate

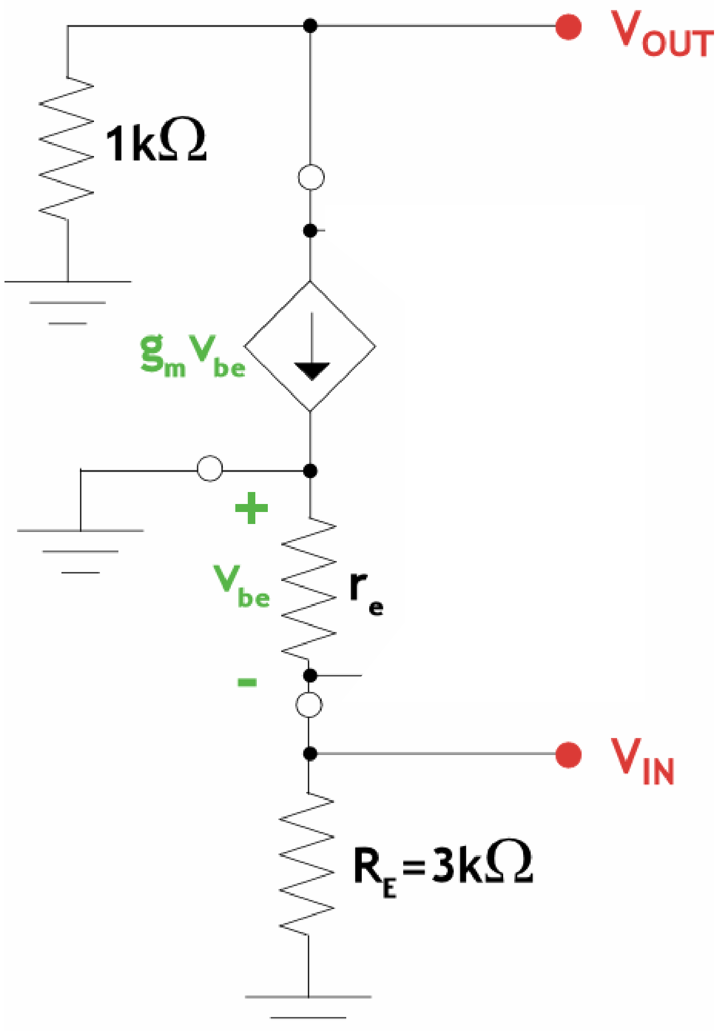
- Moderate gain due to R_E which reduces gain
- Large R_{IN} good for voltage and transconductance amplifier configuration
- Moderate R_{OUT} a problem, but fixable

The Common Base Amplifier (CBA)



- The input is $v_e(t)$
- The output is $v_c(t)$
- The base has a small-signal ground and is "common" to both input and output.
- By-pass capacitors and coupling capacitors are used to DC-bias transistor and couple signals.
- Use T model without the output resistance, r_o.

CBA – Voltage Gain Analysis



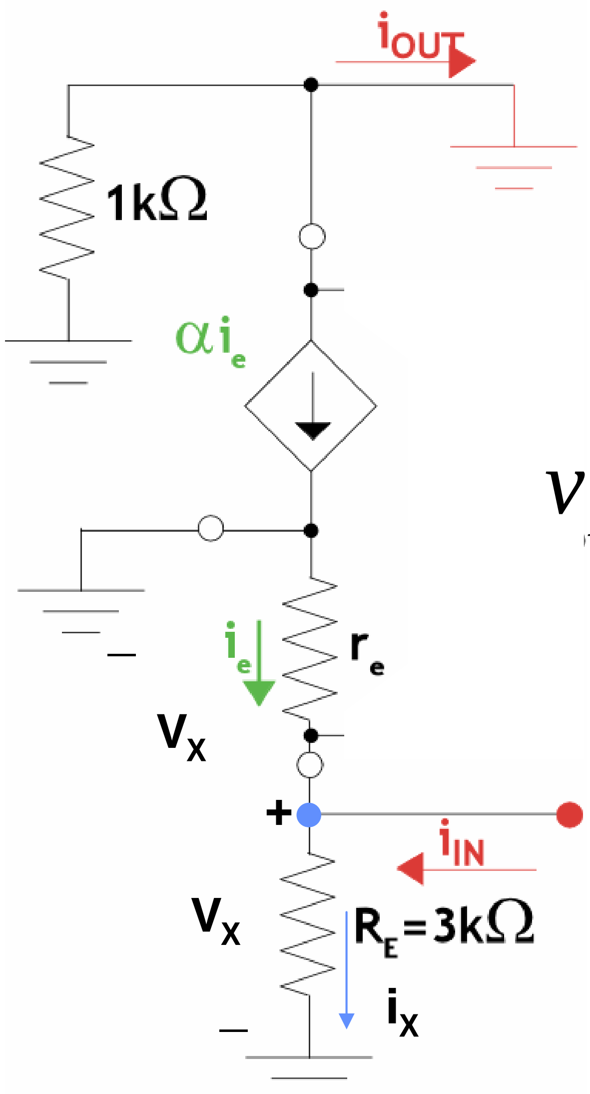
- Draw small-signal T-model equivalent circuit.
- Neglect r_O
- Circuit analysis - voltage gain

$$v_{be} = -v_{in}$$

$$v_{out} = -g_m v_{be}(1k\Omega)$$

$$A_{VO} = \frac{v_{out}}{v_{in}} = g_m(1k\Omega)$$

CBA – Short Circuit Current Gain Analysis



- Draw small-signal T-model equivalent circuit.
- Neglect r_O
- Circuit analysis - current gain

$$v_X = -i_e r_e = i_X R_E \qquad i_X = i_e + i_{in}$$

$$i_e = -\left(\frac{R_E}{r_e + R_E}\right) i_{in} \qquad i_{out} = -\alpha i_e$$

$$\boxed{A_{IS} = \frac{i_{out}}{i_{in}} = \alpha\left(\frac{R_E}{r_e + R_E}\right)}$$

CBA – Input/Output resistance

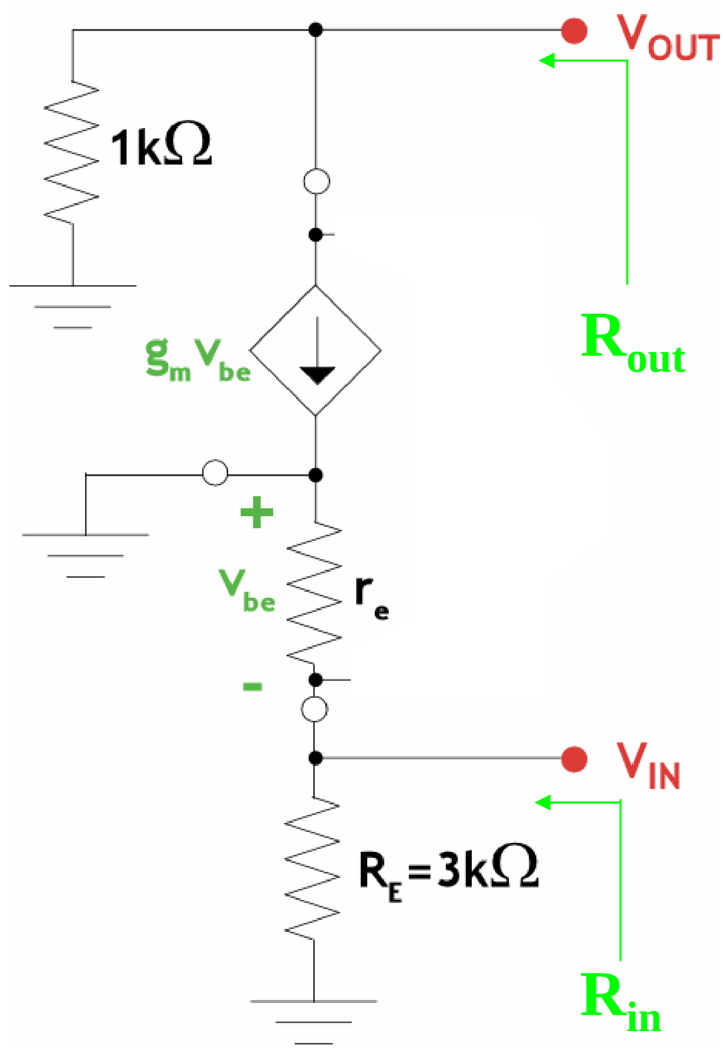
- Input Resistance

$$R_{IN} = r_e \parallel R_E$$

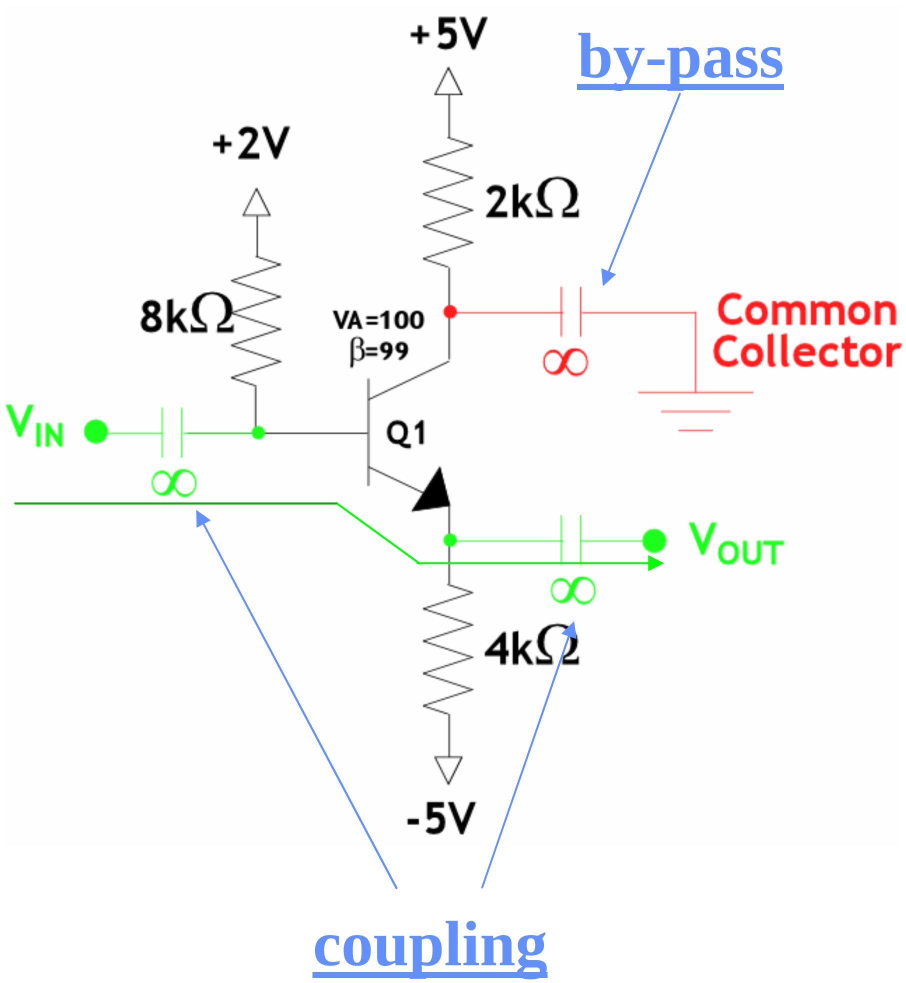
LOW input resistance can be useful !!

- Output resistance
- With $v_{IN} = -v_{be} = 0$

$$R_{OUT} = 1k\Omega$$



The Common Collector Amplifier (CCA)



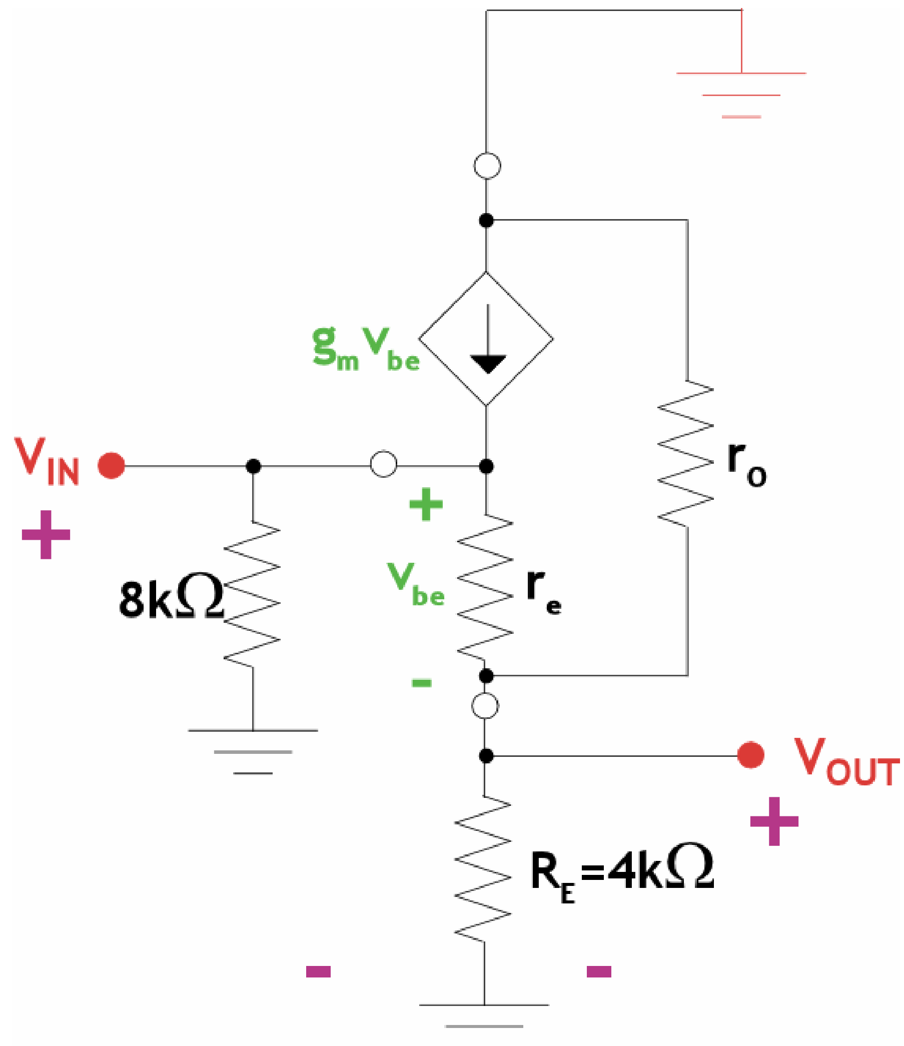
- The input is $v_b(t)$
- The output is $v_e(t)$
- The collector has a small-signal ground and is "common" to both input and output.
- By-pass capacitors and coupling capacitors are used to DC-bias transistor and couple signals.
- Use T model *with* the output resistance, r_o.

CCA – Voltage-Gain Analysis

Voltage Division:

$$v_{out} = v_{in}\left(\frac{r_o // 4k\Omega}{r_o // 4k\Omega + r_e}\right)$$

Since r_e small, Almost $v_{out}=v_{in}$



The CCA – Current-Gain Analysis

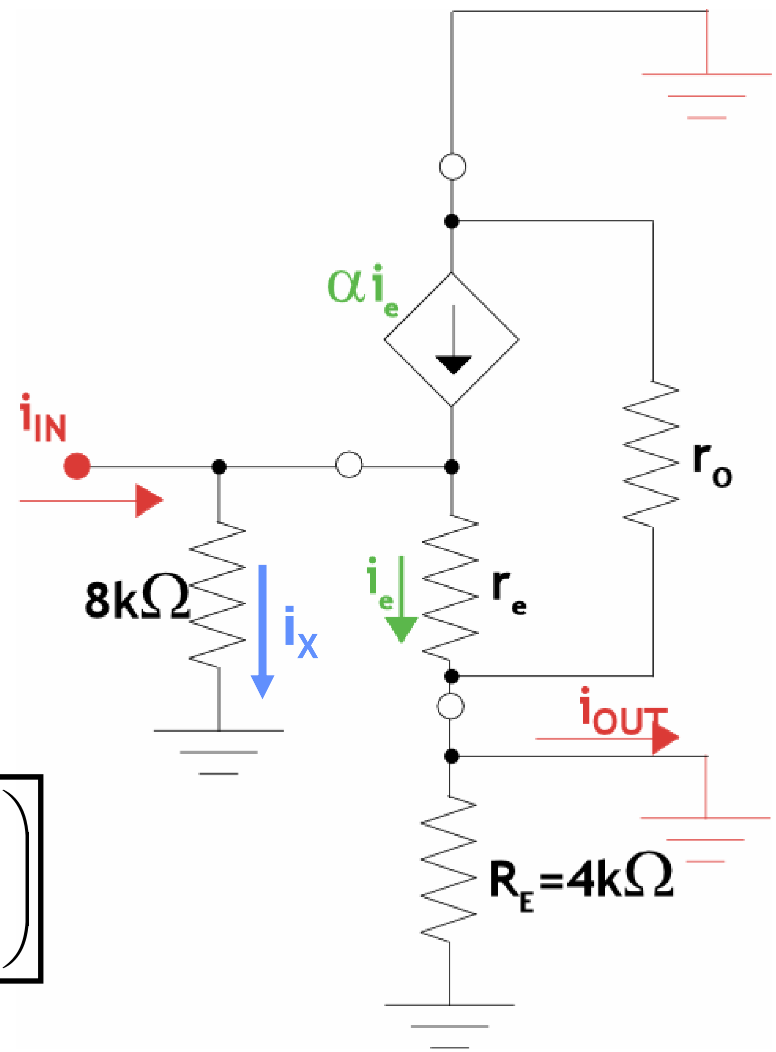
$$i_{out} = i_e$$

$$i_b + \alpha i_e = i_e \ ; \ i_b = i_{in} - i_x$$

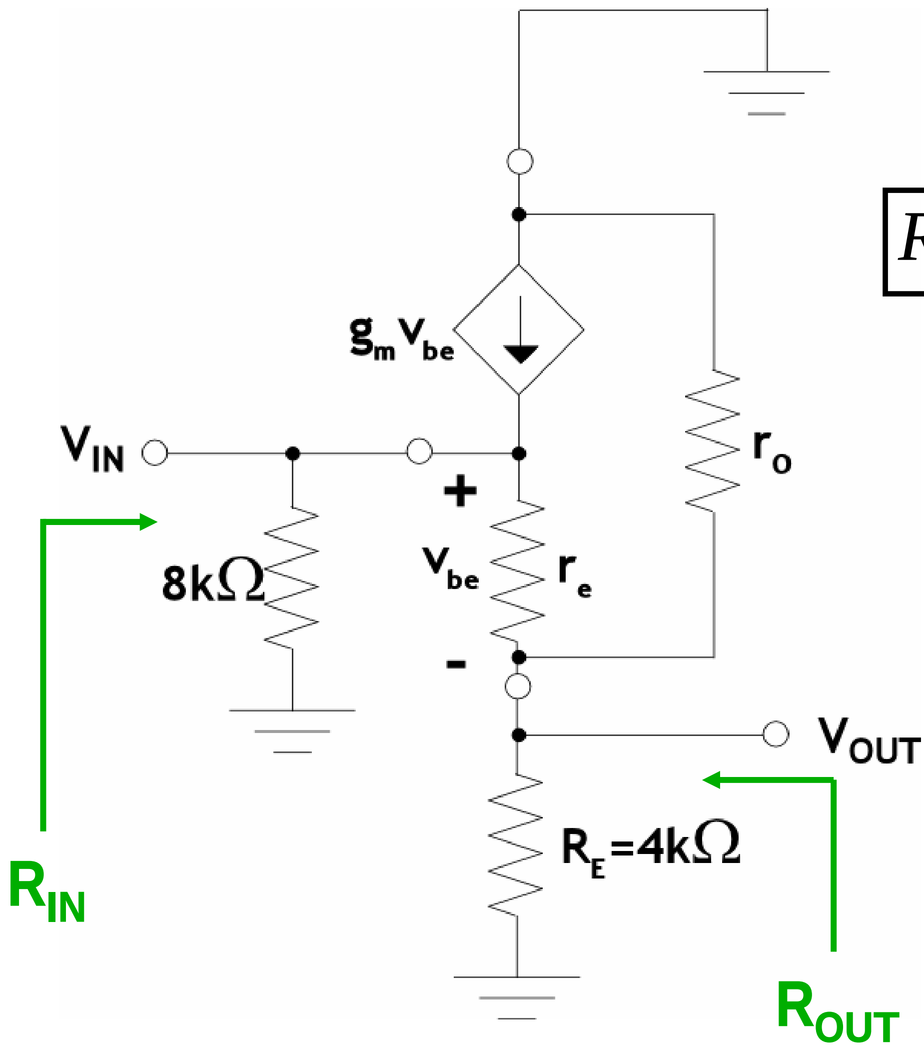
$$i_{in} - i_x = i_e / (1+\beta)$$

$$v_{in} = i_x (8k\Omega) = i_e r_e$$

$$A_{IS} = \frac{i_{out}}{i_{in}} = (\beta + 1)\left(\frac{8k\Omega}{8k\Omega + r_e(\beta+1)}\right)$$



CCA - Input/Output Resistance



- Input Resistance; by inspection using β+1 rule:

$$R_{IN} = 8k\Omega // \{(\beta+1)(r_e + r_o // R_E)\}$$

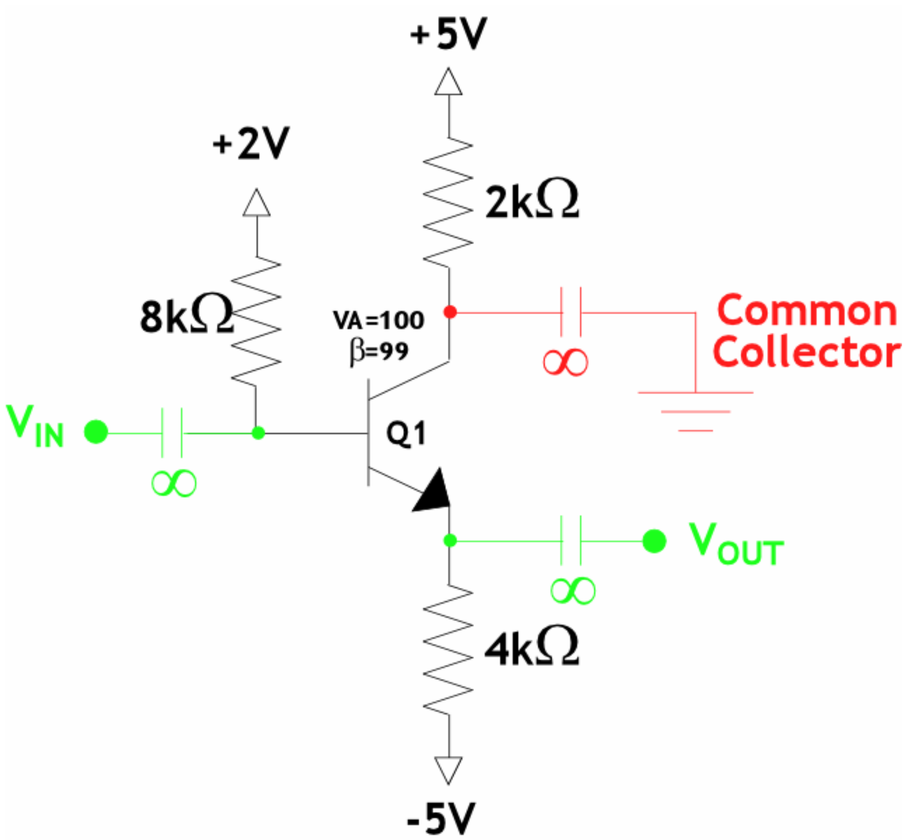
HIGH INPUT RESISTANCE

- Output Resistance, v_{in}= 0

$$R_{OUT} = r_e // r_o // R_E$$

LOW OUTPUT RESISTANCE

CCA Operation – Voltage Buffer



- Good voltage buffer
  - The VOLTAGE gain is almost unity, and the DC component is only reduced by 0.7V
  - Large short circuit current gain
  - High input resistance (which reduces loading to the circuits before)
  - Low output resistance (which reduces the loading of circuits after)