# More MOSFETs from the Text Book

- Current-Voltage Characteristics

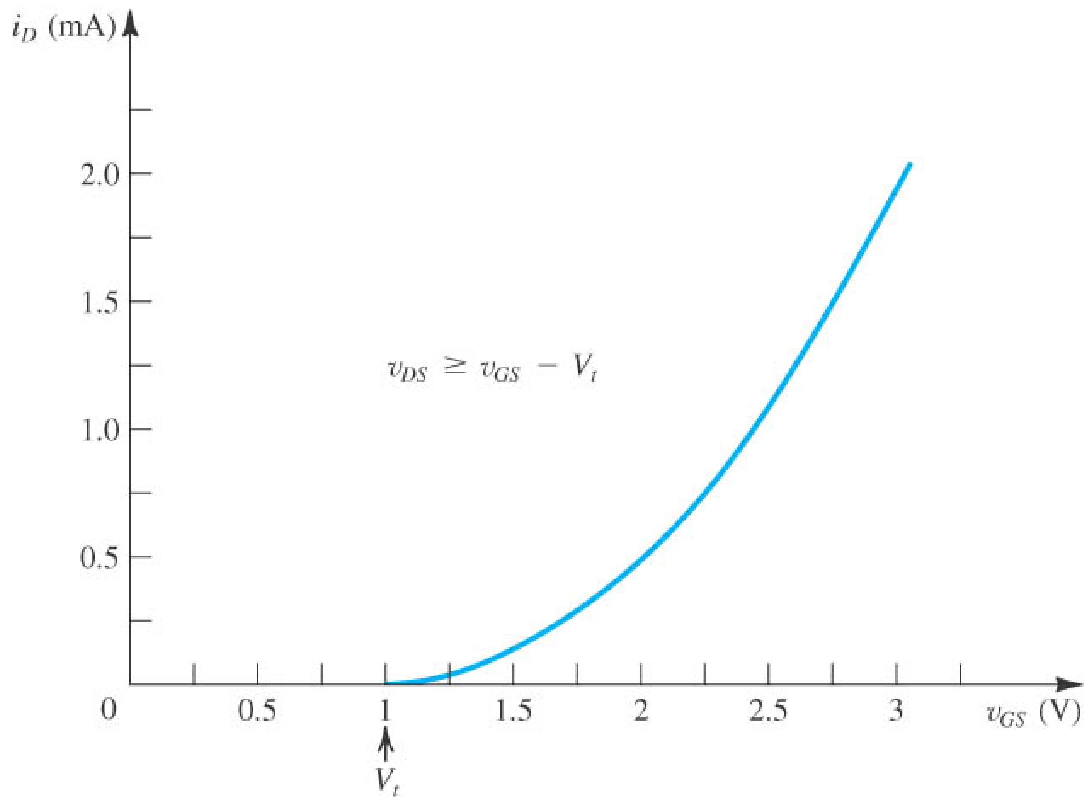


**Figure 4.12** The $i_D$–$v_{GS}$ characteristic for an enhancement-type NMOS transistor in saturation ($V_t$ = 1 V, $k'_n W/L$ = 1.0 mA/V$^2$).

# More MOSFETs from the Text Book

- Circuit representation of Current-Voltage Characteristics in the **Saturation Region** *ignoring channel length modulation effect*

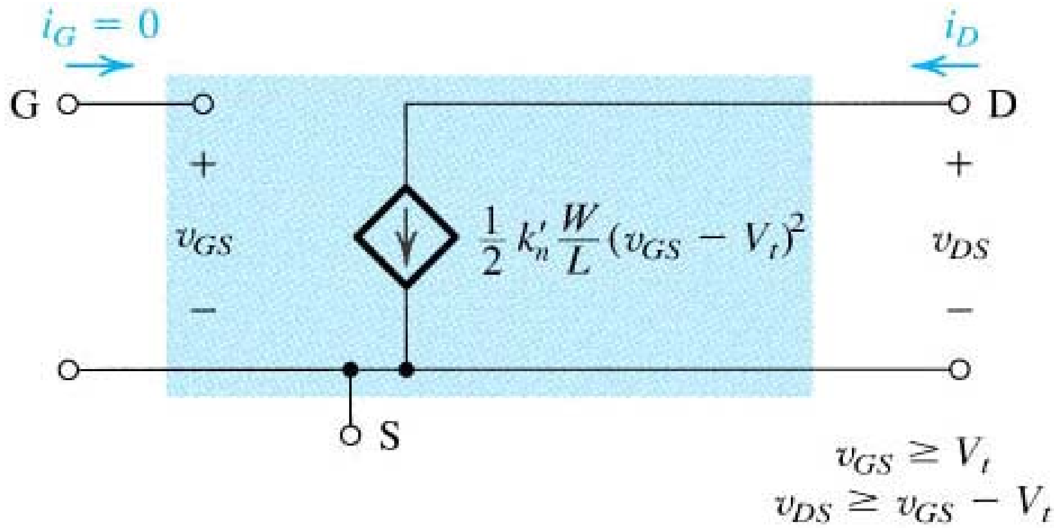


**Figure 4.13** Large-signal equivalent-circuit model of an *n*-channel MOSFET operating in the saturation region.

- This circuit is called **Large-Signal equivalent circuit**
- **Note:** This circuit is **NOT USED** in most of our practical MOSFET problems

# Outline of Chapter 4

- *1- Intro to MOS Field Effect Transistor (MOSFET)*
- 2- NMOS FET
- 3- PMOS FET
- 4- DC Analysis of MOSFET Circuits
- 5- MOSFET Amplifier
- 6- MOSFET Small Signal Model
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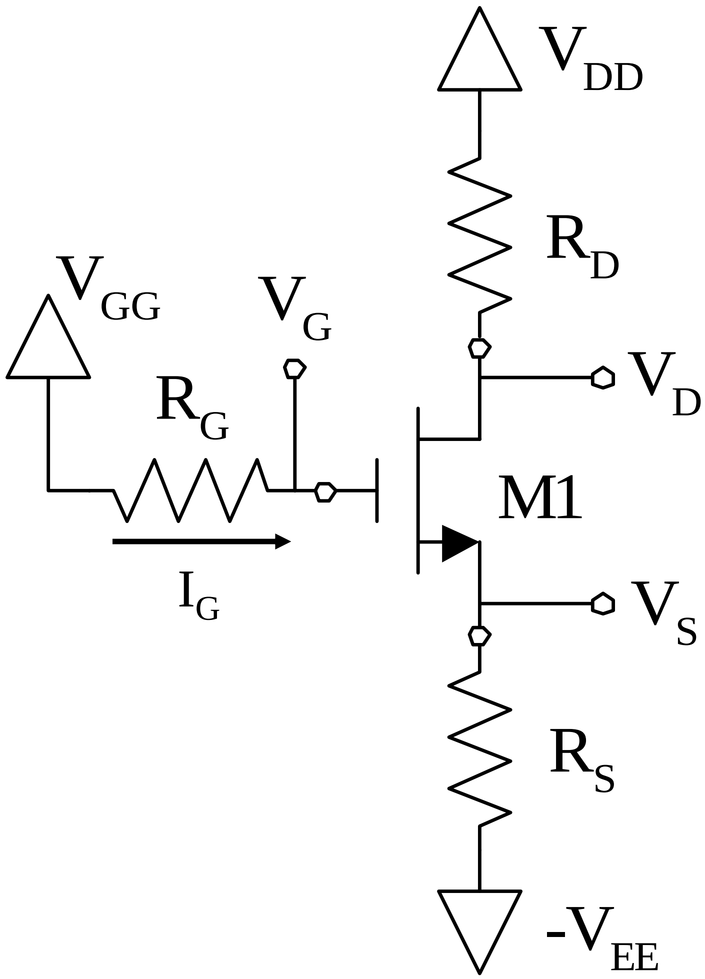
# MOSFET DC Analysis

1. Make an assumption about the state of operation
   - There are three possibilities
   - The mostly used case is saturation mode
   - This is the mode used in designing amplifiers
2. Solve to find DC voltages and currents
   - Namely, $I_D$, $V_D$, $V_S$, and $V_G$
3. Verify the found values with the inequalities describing the state of operation
4. If not correct, start solving the circuit again with another assumption about the state of operation

# Example 1




- No gate current: $I_G = 0 \Rightarrow V_G = V_{GG}$
- Have expressions for $I_D$:

$$I_D = \frac{V_{DD} - V_D}{R_D} = \frac{V_S + V_{EE}}{R_S}$$

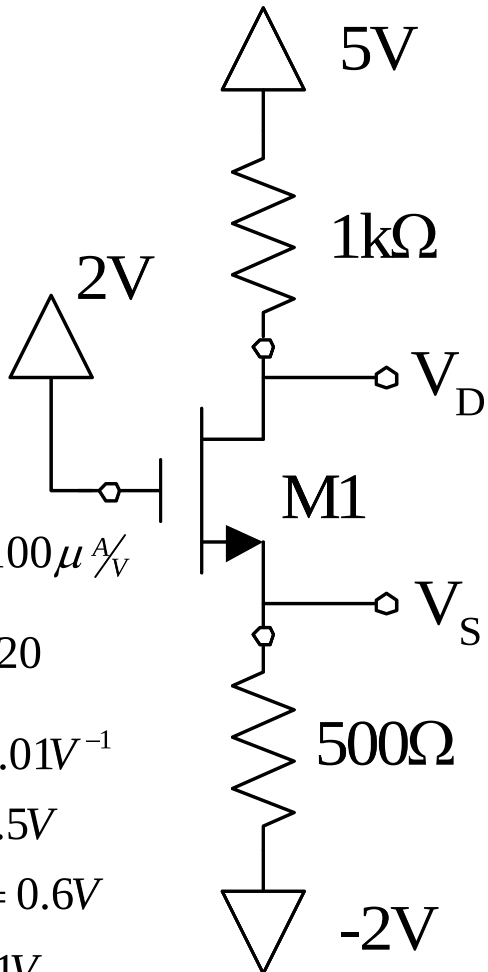
– Assume saturation mode:

$$I_D = \tfrac{1}{2} k_n' \frac{W}{L} \left(V_{GS} - V_t\right)^2 \left(1 + \lambda V_{DS}\right)$$

– Body effect: $V_B = -V_{EE} \Rightarrow V_S \neq V_B$

$$V_t = V_{t0} + \gamma\left(\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}\right)$$

# Example 1 – con't




$k'_n = 100\mu A/V$

$\frac{W}{L} = 20$

$\lambda = 0.01V^{-1}$

$\gamma = 0.5V$

$2\phi_f = 0.6V$

$V_{t0} = 1V$

$$I_D = \frac{5 - V_D}{1k} = \frac{V_S + 2}{500}$$

- **<u>Assume saturation</u>** mode:

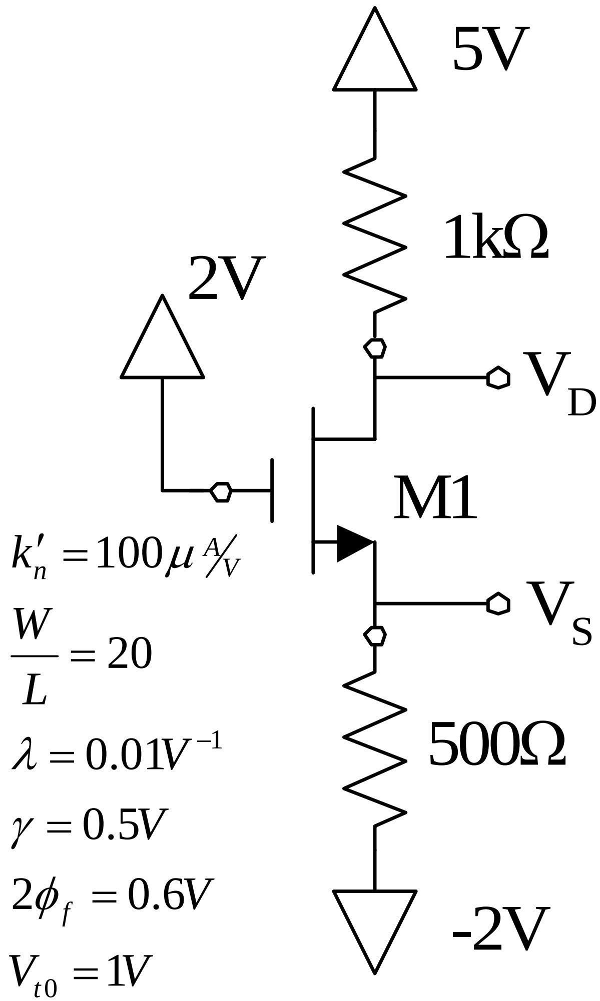
$$I_D = 1m(2 - V_S - V_t)^2[1 + \lambda(V_D - V_S)]$$

$$V_t = 1 + 0.5\left(\sqrt{0.6 + V_S + 2} - \sqrt{0.6}\right)$$

- With $V_D$ in terms of $V_S$, get cubic $I_D$ expression
- Neglect CLM and Body Effect.

$$I_D = 1m(2 - V_S - 1)^2$$

# Example 1 – con't




$$I_D = 1m(2 - V_S - 1)^2 = \frac{V_S + 2}{500}$$

$$V_S^2 - 4V_S - 3 = 0$$

$$V_S = 4.646V \ \ or \ \ V_S = -0.646V$$

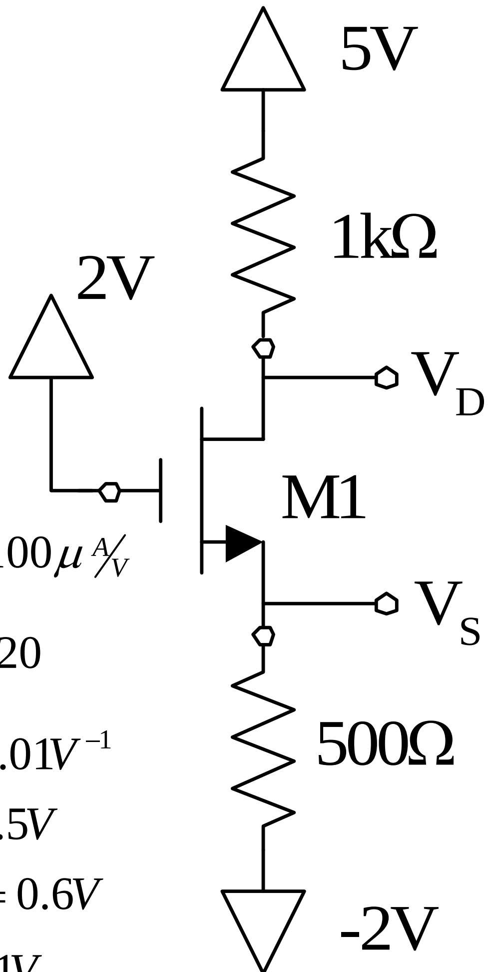
**Check conditions for saturation**

$$V_{GS} > V_t \Rightarrow 2 - V_S > 1 \Rightarrow V_S < 1V$$

$$\boxed{V_S = -0.646V}$$

$$\boxed{I_D = 2.708mA}$$

# Example 1 – con't




$k_n' = 100\mu A/V$

$\frac{W}{L} = 20$

$\lambda = 0.01V^{-1}$

$\gamma = 0.5V$

$2\phi_f = 0.6V$

$V_{t0} = 1V$

$$V_D = 5 - I_D R_D = \underline{2.292V}$$

- **Verify saturation** :

$$V_{DS} > V_{GS} - V_t$$

$$V_D - V_S > V_G - V_S - V_t$$

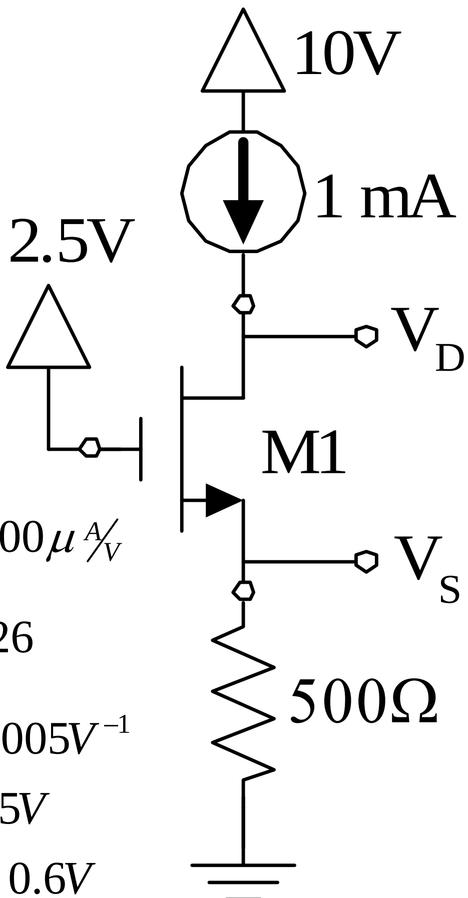
$$2.292 > 2 - 1 = 1$$

- Maximum $R_D$ that sustains saturation mode: $V_D > 1V$?

$$V_D = 5 - (2.708m)R_D > 1V$$

$$\boxed{R_D < 1.477k\Omega}$$

# Example 2




$k_n' = 100\mu A/V$

$\frac{W}{L} = 26$

$\lambda = 0.005V^{-1}$

$\gamma = 0.5V$

$2\phi_f = 0.6V$

$V_{t0} = 1V$

$$I_D = 1mA = \frac{V_S}{500} \Rightarrow V_S = 0.5V$$

- Calculate $V_t$:

$$V_t = 1 + 0.5\left(\sqrt{0.6+0.5} - \sqrt{0.6}\right) = 1.137V$$

- **<u>Assume saturation mode</u>**:

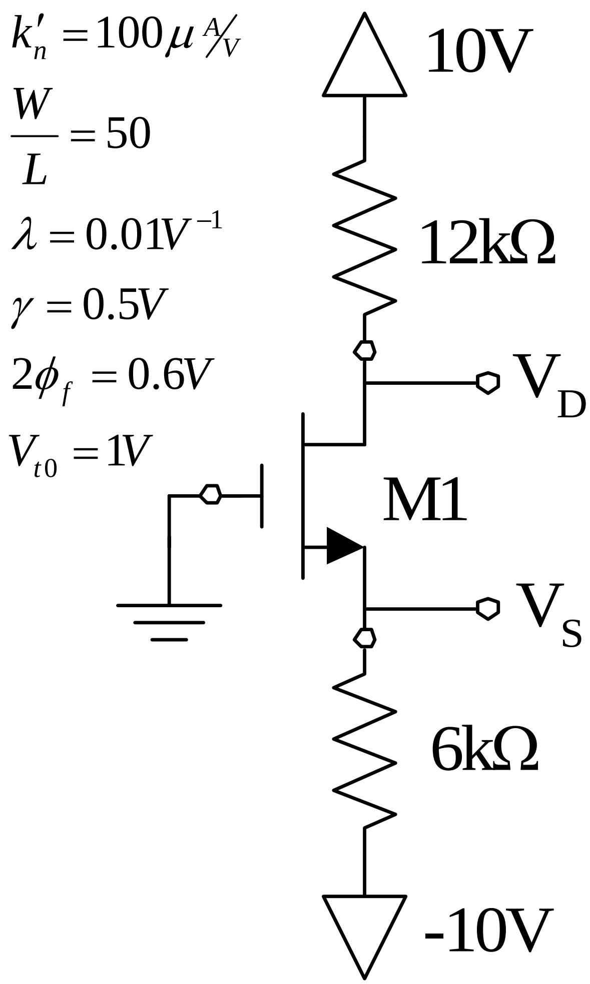
$$I_D = 1.3m(2.5 - 0.5 - 1.137)^2\left[1 + 5m(V_D - 0.5)\right]$$

- Solve for $V_D$:

**Verify the assumption**

$$\boxed{V_D = 7.07V} > V_G - V_t$$

# Example 3




$$I_D = \frac{10 - V_D}{12k} = \frac{V_S + 10}{6k}$$

- **<u>Assume saturation mode</u>**:

$$I_D = 2.5m(-V_S - V_t)^2[1 + \lambda(V_D - V_S)]$$

- No simple way to find $V_S$; neglect Body effect
- With $V_D$ in terms of $V_S$, get cubic for $I_D$; neglect CLM

$$I_D = 2.5m(-V_S - 1)^2 = \frac{V_S + 10}{6k}$$

$$15 \cdot V_S^2 + 29 \cdot V_S + 5 = 0$$

$$V_S = -0.191V \quad or \quad V_S = -1.742V$$

# Example 3 – con't

$k_n' = 100\mu A/V$

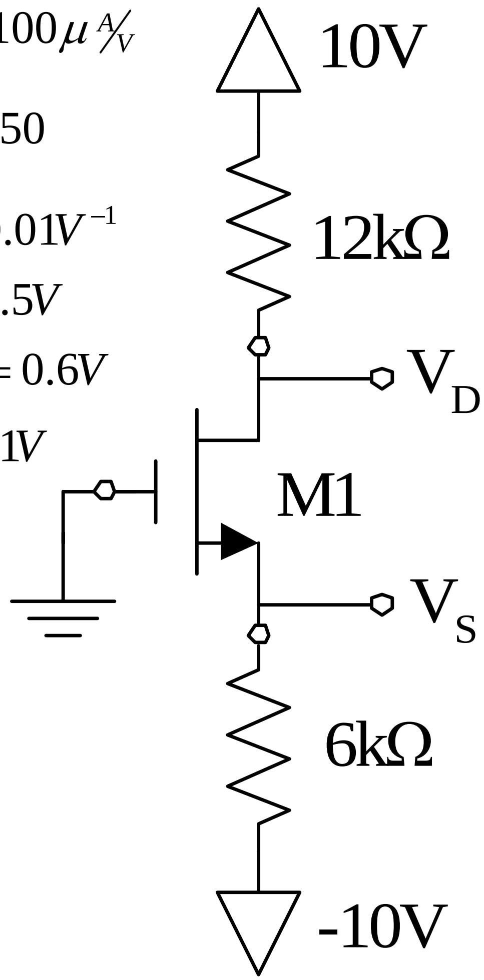
$\frac{W}{L} = 50$

$\lambda = 0.01V^{-1}$

$\gamma = 0.5V$

$2\phi_f = 0.6V$

$V_{t0} = 1V$




- Require $V_{GS} > V_t$ $\boxed{V_S = -1.742V}$
- Solve for $I_D$: $I_D = \frac{V_S + 10}{6k} = \underline{1.376mA}$
- Solve for $V_D$: $V_D = 10 - I_D R_D$
  $= \underline{-6.51V}$

But, require $V_D > V_G - V_t = -1V$

$\therefore$ ***Cannot be in saturation***

- <u>Assume triode region</u> :

$$I_D = 5m\left[\left(V_{GS} - V_t\right)V_{DS} - \tfrac{1}{2}V_{DS}^2\right]$$

Need expression for $V_{DS}$

# Example 3 – con't

$$I_D = \frac{10 - V_D}{12k} = \frac{V_S + 10}{6k} \qquad V_D = -2V_S - 10 \qquad V_{DS} = -3V_S - 10$$

$k_n' = 100\mu A/V$

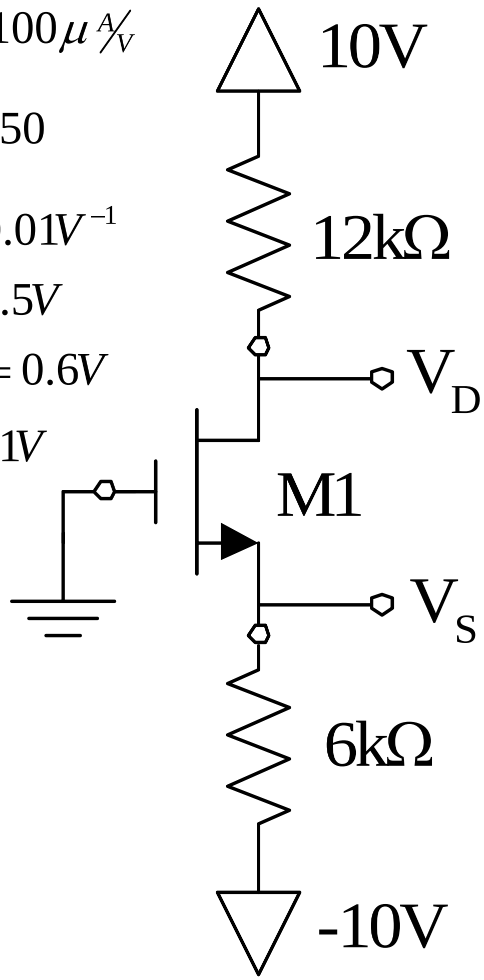
$\frac{W}{L} = 50$

$\lambda = 0.01V^{-1}$

$\gamma = 0.5V$

$2\phi_f = 0.6V$

$V_{t0} = 1V$




- Substitute & solve for $V_S$:

$$45 \cdot V_S^2 + 511 \cdot V_S + 1210 = 0$$

$$V_S = -3.365V \quad or \quad V_S = -7.991V$$

- Find corresponding $V_D$'s:

$$V_D = -3.27V \quad or \quad V_D = 5.982V$$

But, require $V_D < V_G - V_t = -1V$

$$V_D = -3.27V \quad \& \quad V_S = -3.37V$$
$$I_D = 1.11mA$$

# DC Analysis – General Comments

What should we do in our engineering calculations?

- CLM generally has small effect at DC (a few percent)
- Can usually be safely ignored
- Neglecting Body effect can result in significant errors in DC analysis
- Rules of thumb (DC Analysis):
  - Ignore CLM unless easy to include
  - Include Body effect unless $V_S$ not easy to find

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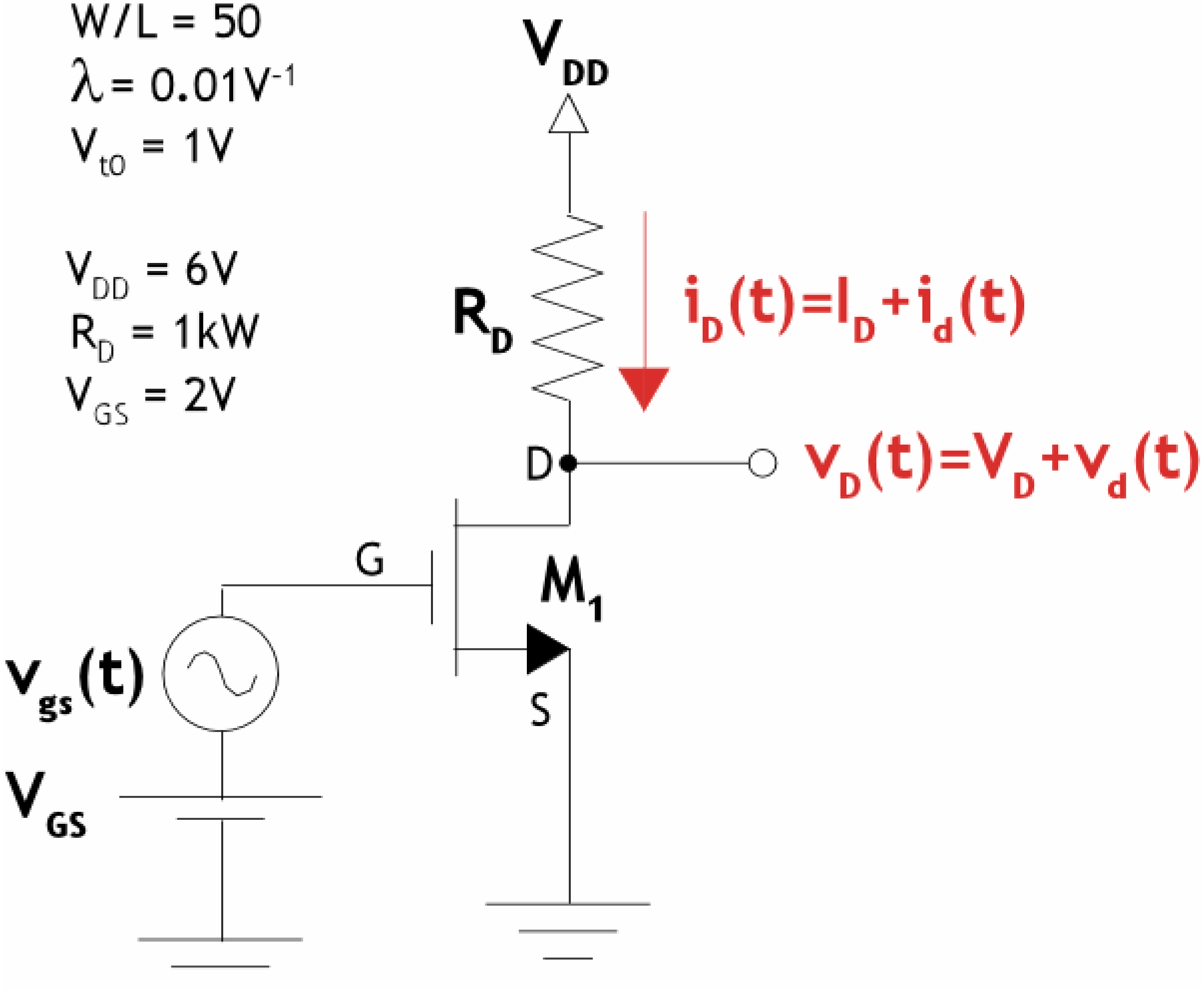
# Small Signal Analysis

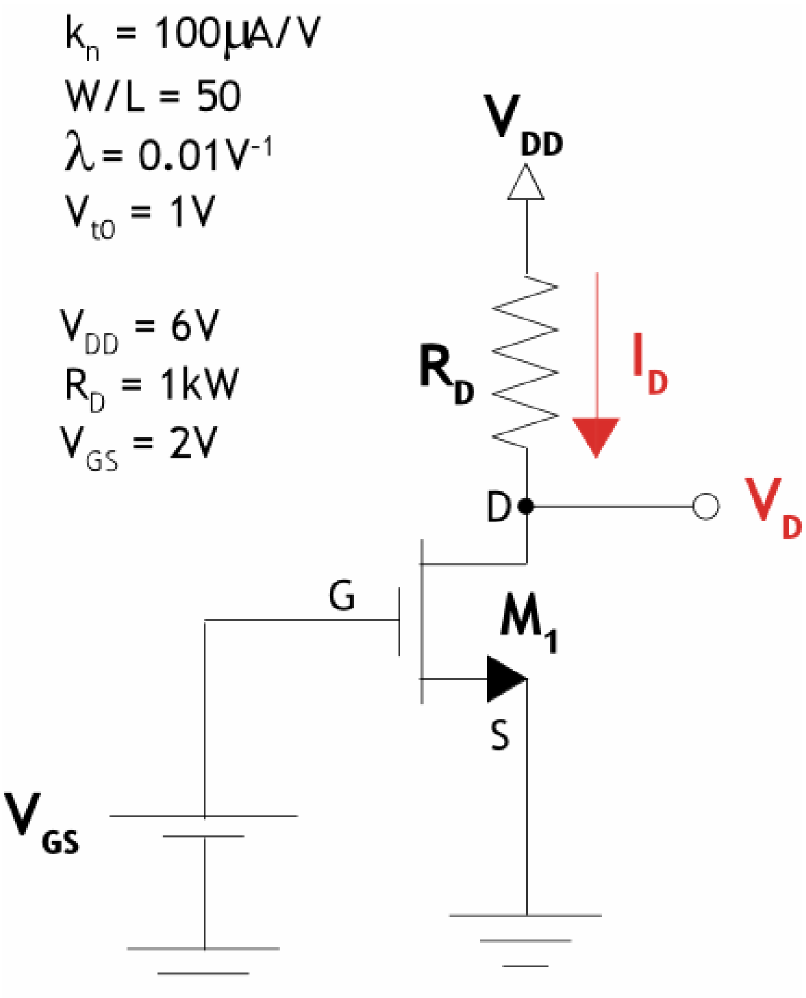
$k_n = 100\mu A/V$
$W/L = 50$
$\lambda = 0.01V^{-1}$
$V_{t0} = 1V$

$V_{DD} = 6V$
$R_D = 1k\Omega$
$V_{GS} = 2V$

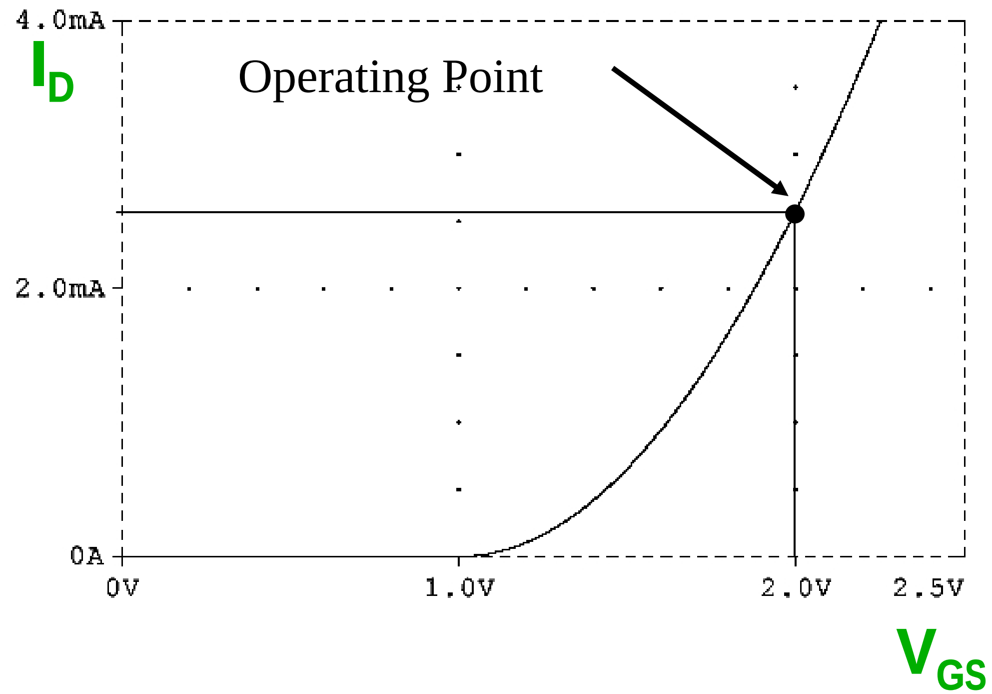



- Gate input has DC and AC components
- So will the drain voltage and current
- Develop small signal model

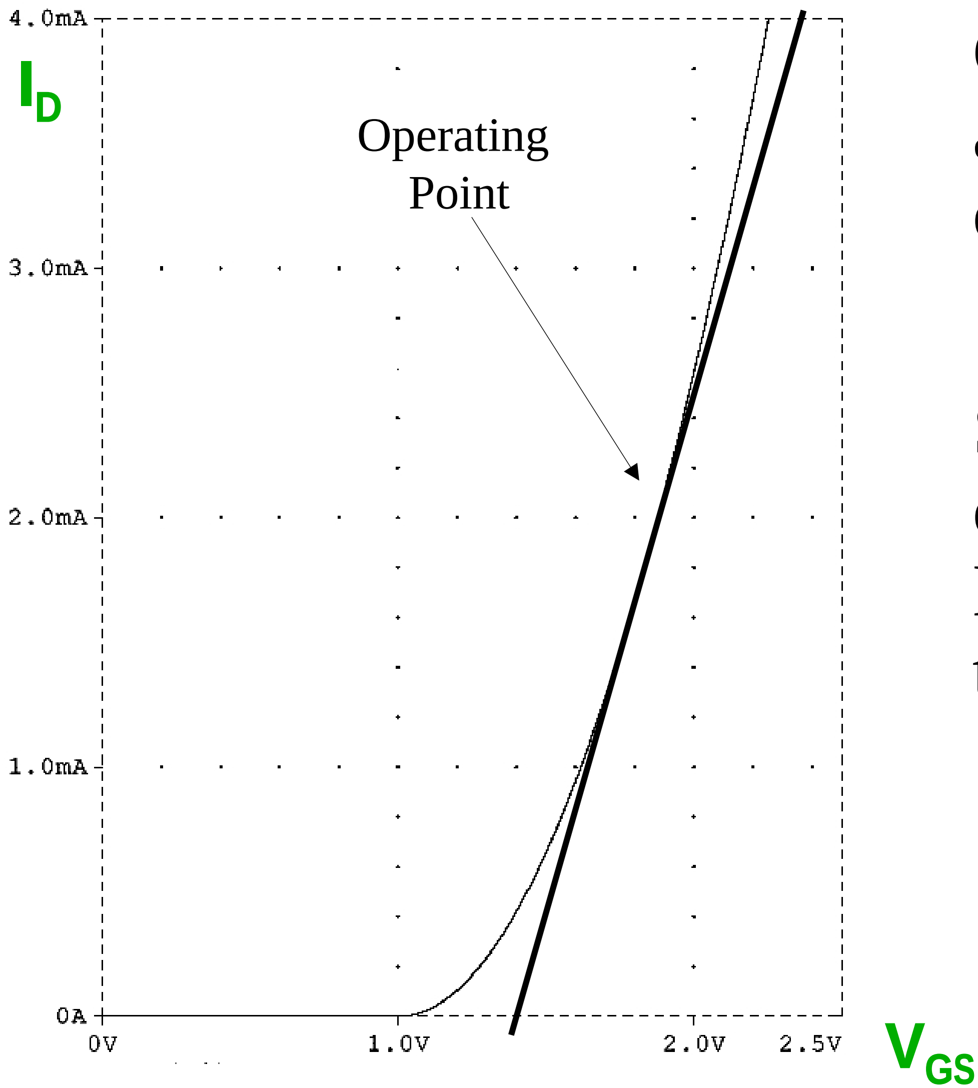
# Operating Point $I_D$ vs $V_{GS}$




- DC analysis, AC sources off
- Operating point determined by $V_{GS}$-$V_t$ in saturation



# Small Signal Superposition – $I_D$ vs $V_{GS}$




Consider superposition of an AC signal at the DC operating point

Slope of $i_d$-$v_{gs}$ curve at operating defined as MOSFET transconductance, $g_m$

$$g_m \equiv \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{OP} \approx \frac{\Delta I_D}{\Delta V_{GS}}$$

# MOSFET Transconductance – $g_m$

- To derive an expression for $g_m$:

$$g_m \equiv \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{i_D = I_D}$$

- Start with full $I_D$ equation

$$i_D = \tfrac{1}{2} k_n' \frac{W}{L} \left(v_{GS} - V_t\right)^2 \left(1 + \lambda \cdot V_{DS}\right)$$

- Take derivative and simplify

$$\frac{\partial i_D}{\partial v_{GS}} = \underbrace{k_n' \frac{W}{L} \left(V_{GS} - V_t\right)\left(1 + \lambda \cdot V_{DS}\right)}_{\dfrac{2 \cdot I_D}{\left(V_{GS} - V_t\right)}}$$

$$\boxed{g_m = \frac{2 \cdot I_D}{V_{GS} - V_t}}$$

# Summary of $g_m$

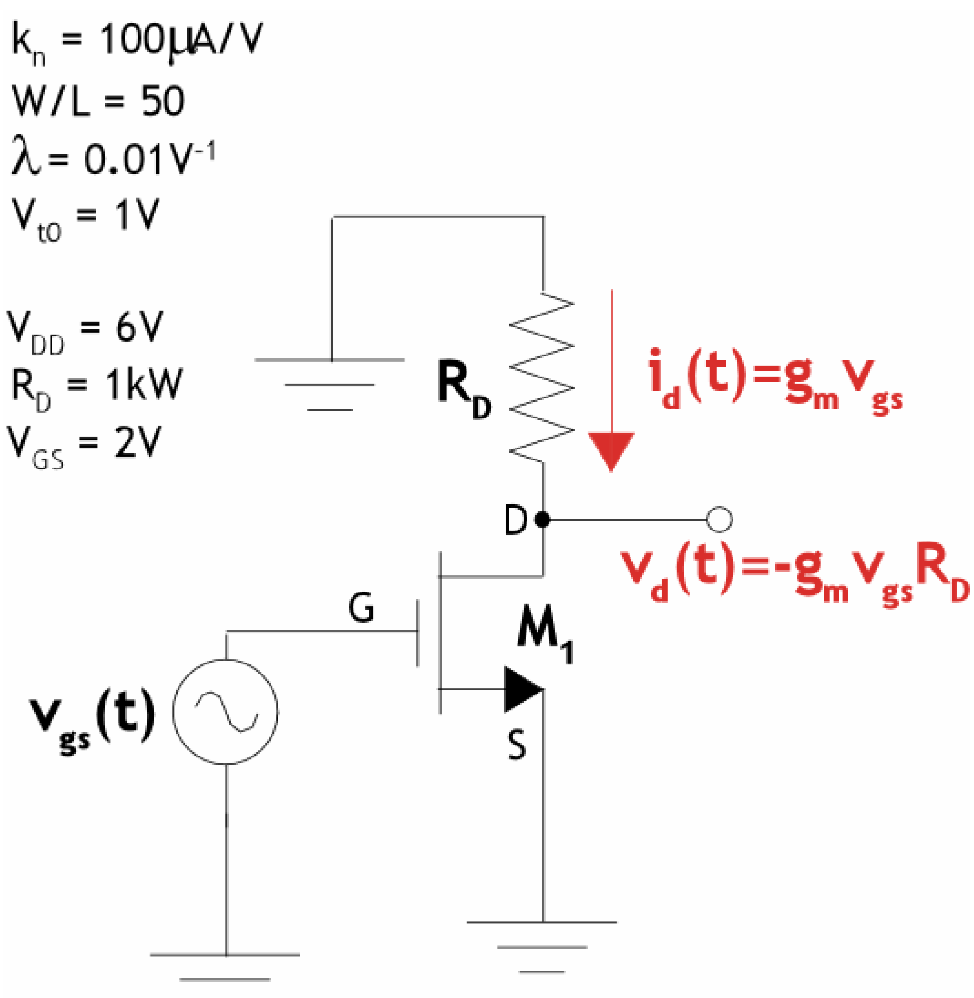
- Sedra & Smith use several expressions for $g_m$

$$g_m = \frac{2 \cdot I_D}{V_{GS} - V_t}$$

$$g_m = k_n' \frac{W}{L}\left(V_{GS} - V_t\right)\left(1 + \lambda \cdot V_{DS}\right)$$

$$g_m = \sqrt{2k_n'}\sqrt{\frac{W}{L}}\sqrt{1 + \lambda \cdot V_{DS}}\sqrt{I_D}$$

# Basic MOSFET Amplifier Operation




- Apply small signal at gate: $v_{gs}$
- Results in signal current flow at drain $i_d$; $g_m$ proportionality constant
- Signal current through $R_D$ produces output voltage
- Output signal voltage equal to: $v_d = -g_{mv}v_{gs}R_D$