Section 3
The pn Junction and Diodes

Sedra/Smith, Sections 3.1-3.7

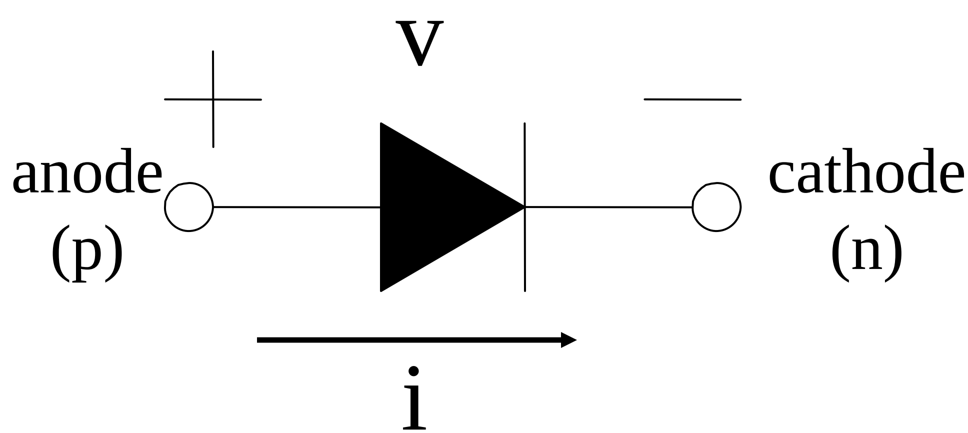
Outline of Section 3 - Diodes

- Other two terminal devices
- Diode models
- Exponential model
- Constant voltage drop model
- Reverse breakdown
- Applications
- Small-signal model
- PN junctions

Resistors, Capacitors, Inductors

- Resistor: R V=IR
- Capacitor: C $$i = c\frac{dv}{dt}$$
- Inductor: I $$v = l\frac{di}{dt}$$
- Devices are two terminals and do not have a required orientation.

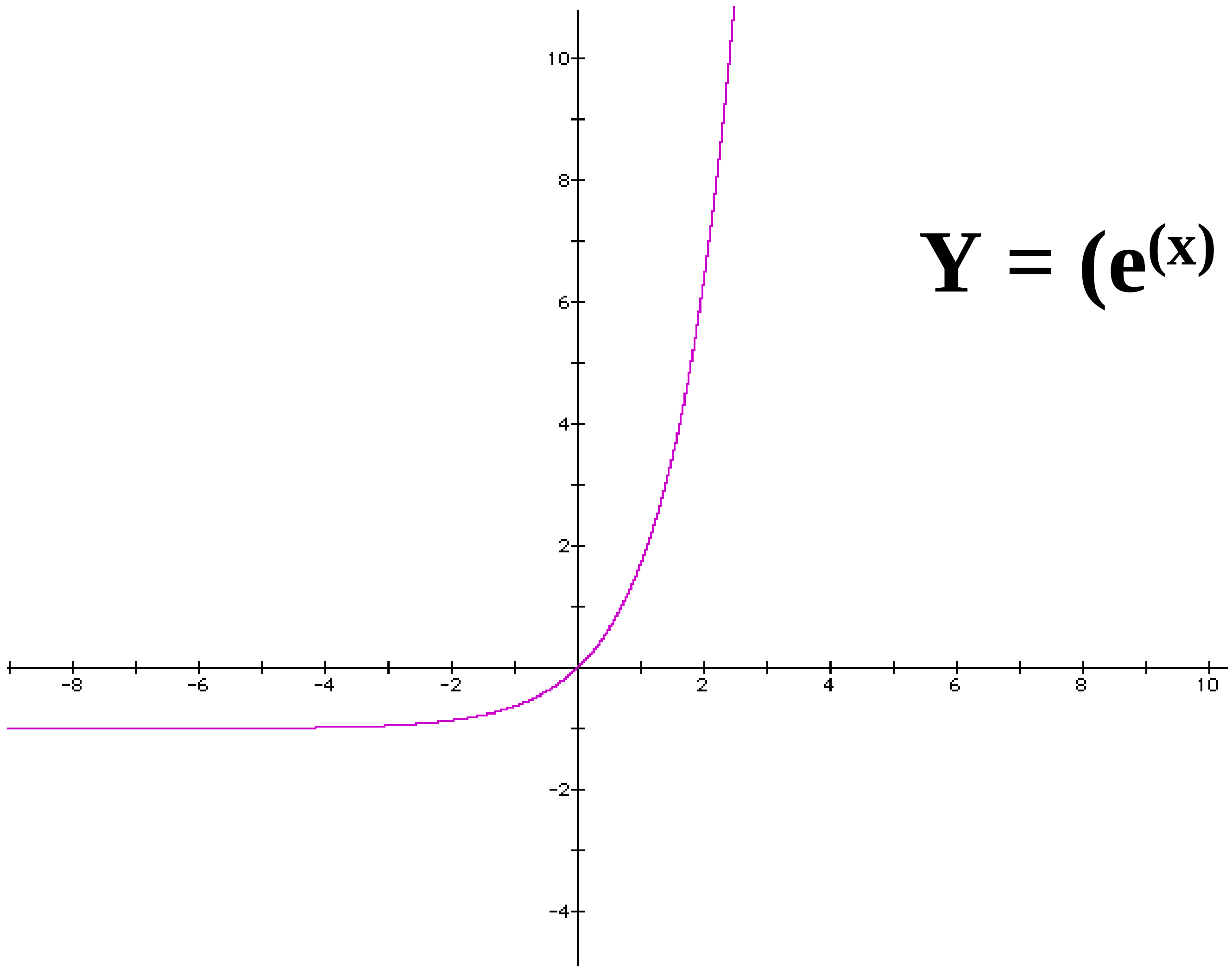
Diode Symbol and Terminal Characteristics



Exponential i-v relation: $i = I_S\left(e^{\frac{v}{nV_T}} - 1\right)$

Exponential Model

Exponential Characteristic Equation



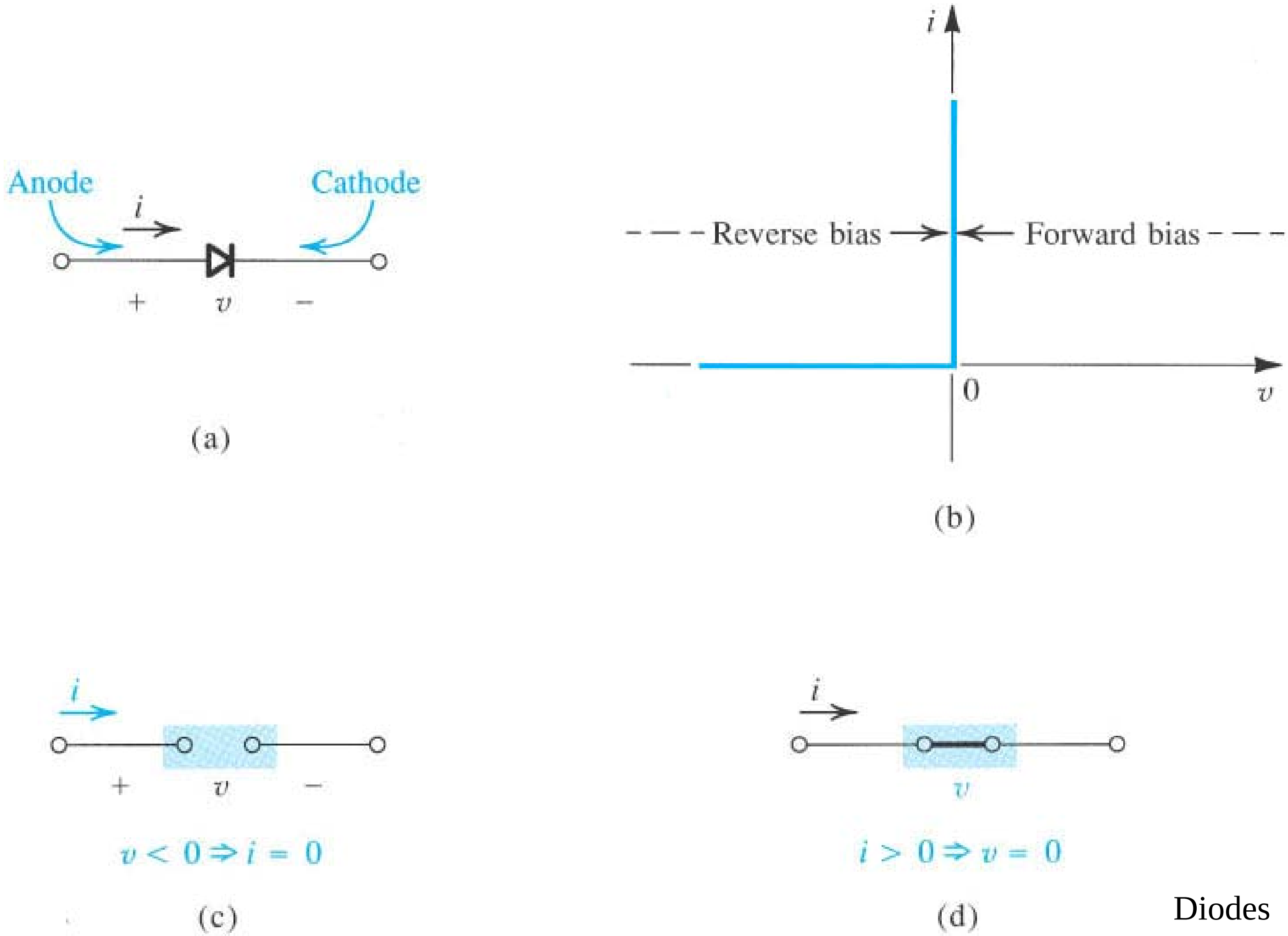
$$Y = (e^{(x)} - 1)$$

Diodes

- It is a nonlinear device
- How to model the nonlinear behavior?
  - Ideal model
  - Exponential model
  - Constant voltage drop model
  - Piecewise-linear (we don't work with this model much, except for Zener diode)

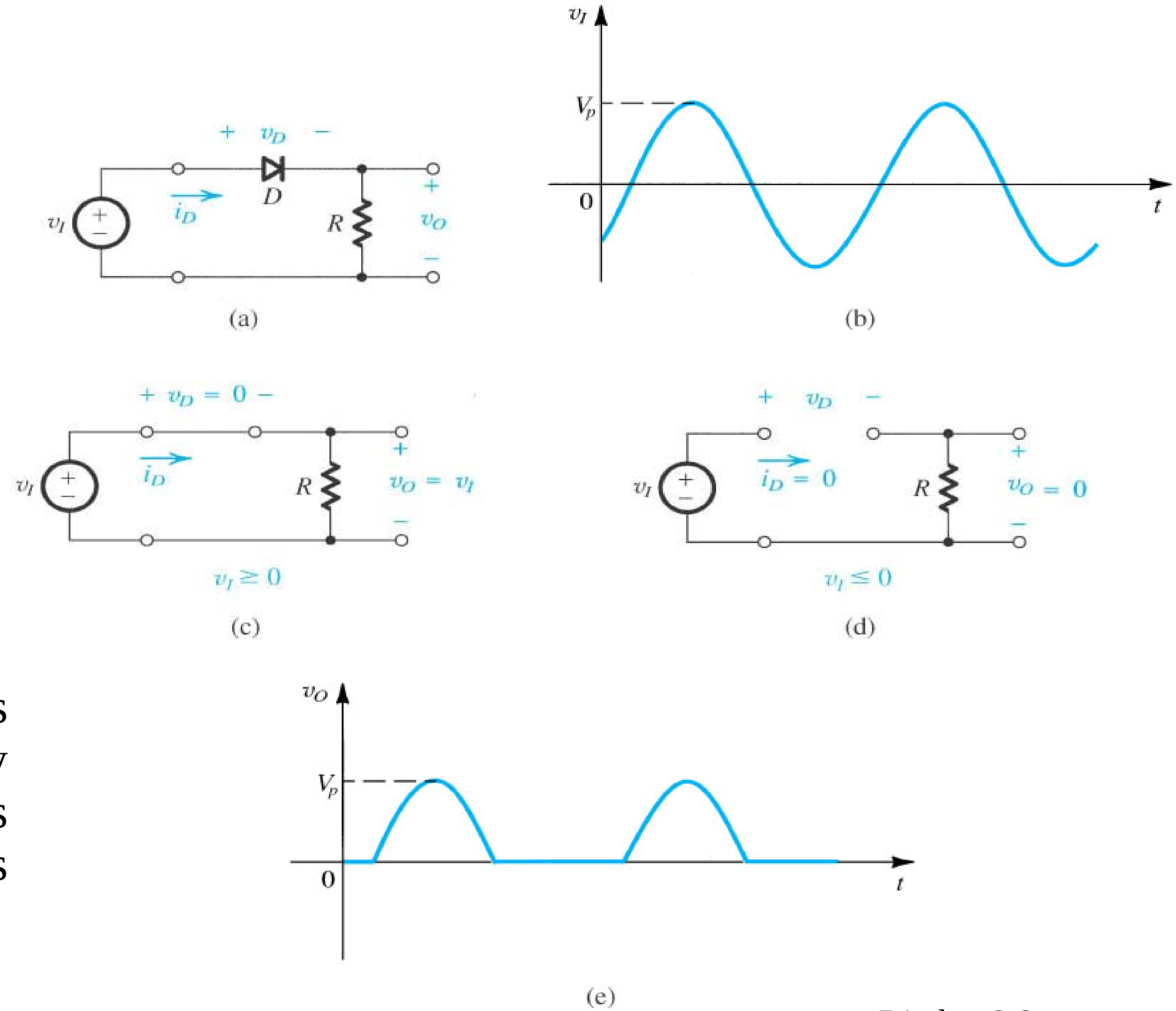
Ideal Model

- Diode is considered to be an ideal switch
  - Used for fast and approximate analysis



Ideal Model Application

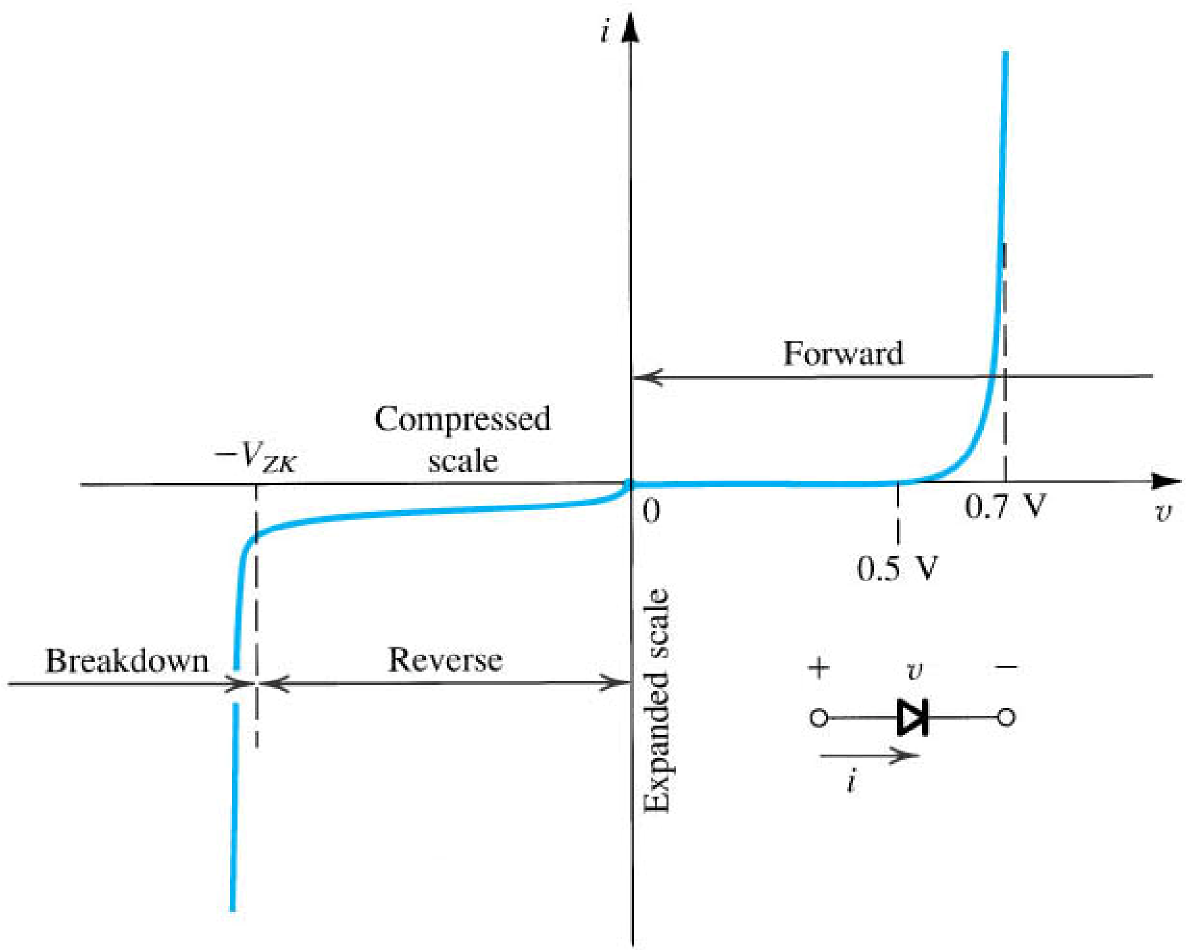
- Example: Simple rectifier circuit
  - We will see a more accurate analysis of this circuit later
- Example: Logic gates
  - This model is actually very useful in analysis of logic circuits and is often used



Outline of Section 3 - Diodes

- Other two terminal devices
- Diode models
- <u>Exponential model</u>
- Constant voltage drop model
- Reverse breakdown
- Applications
- Small-signal model
- PN junctions

I-V Characteristic of a Diode



This nonlinear i-v characteristic can be described for most of its parts with the Exponential Model

Exponential Model Definitions

Exponential Model: $i = I_S\left(e^{\frac{v}{nV_T}} - 1\right)$

- **I_S: reverse saturation current**
  - proportional to cross-sectional area of current flow
  - discrete Si devices: $I_S \sim 10^{-9}$-10^{-13} A
  - IC Si devices: $I_S \leq 10^{-15}$ A
- **n: fitting parameter**
  - normally between 1 and 2 for Si
  - discrete Si devices: n ~ 2
  - IC Si devices: n ~ 1

- **V_T: Thermal Voltage**
  - from device physics:

$$V_T = \frac{k \cdot T}{q}$$

  - **k: Boltzmann constant** ($1.38\text{x}10^{-23}$ J/K)
  - **T: Temperature** (Kelvin)
  - **q: electron charge** ($1.6\text{x}10^{-19}$ C)
  - At room temperature, **V_T ~ 25 mV**

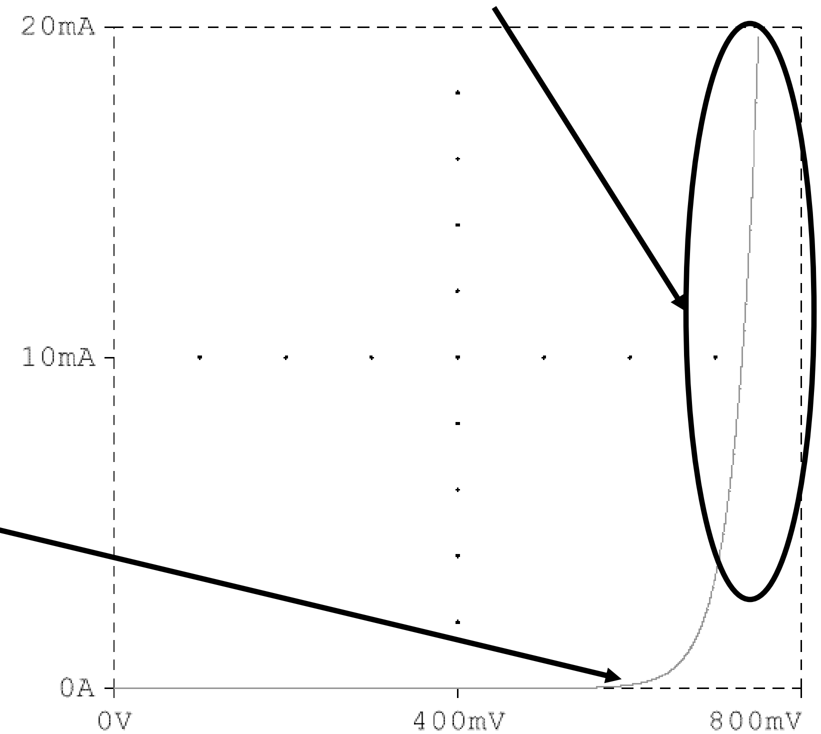
Exponential Model – Forward Bias

As V increases, $\exp\left(\frac{v}{n \cdot V_T}\right) >> 1$

$$i \cong I_S e^{\frac{v}{nV_T}}$$

- The voltage at which the diode starts to conduct appreciably is called the ***cut-in voltage;*** value is ≈ .5V for silicon diodes

When diode is fully conducting, V remains constant at ~ 0.7V for silicon diodes



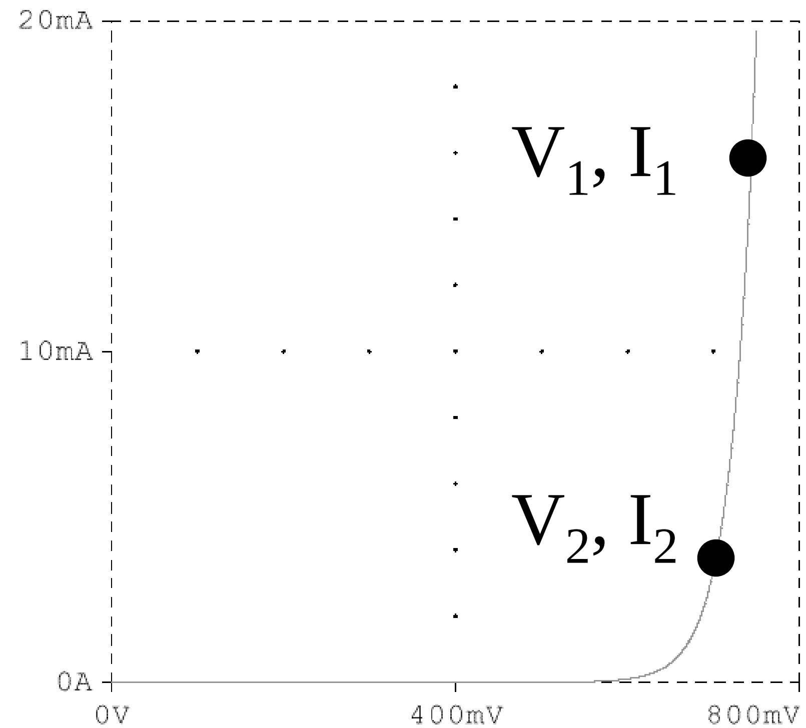
Forward Bias Analysis

$$i = I_S e^{\frac{v}{nV_T}} \quad \Longleftrightarrow \quad v = nV_T \ln\left(\frac{i}{I_S}\right)$$

Consider two points on I-V curve above cut-in voltage: (V_1, I_1) and (V_2, I_2)

$$\frac{I_2}{I_1} = \exp\left(\frac{V_2 - V_1}{n \cdot V_T}\right)$$

$$V_2 - V_1 = n \cdot V_T \cdot \ln\left(\frac{I_2}{I_1}\right)$$



Strong Forward Bias

- Given: a diode with n = 1 and I = 1mA at V = 0.7V
- Question: determine the *voltage drop* across diode when the *current* flowing through the diode is doubled:

$$V_2 - 0.7 = n \cdot V_T \cdot \ln\left(\frac{2mA}{1mA}\right)$$

$$V_2 - 0.7 = (1)(25mV)\ln(2)$$

$$V_2 = 0.717V$$

- I-V data points for n = 1 and $I_S = 6.9\text{x}10^{-16}$A:

I	V
1pA	0.180V
10pA	0.239V
100pA	0.297V
1nA	0.355V
10nA	0.412V
100nA	0.470V
1µA	0.527V
10µA	0.585V
100µA	0.642V
1mA	0.700V
10mA	0.758V
100mA	0.815V

Note from data, above 10mA, a 10X increase in I results in only a 57mV increase in V

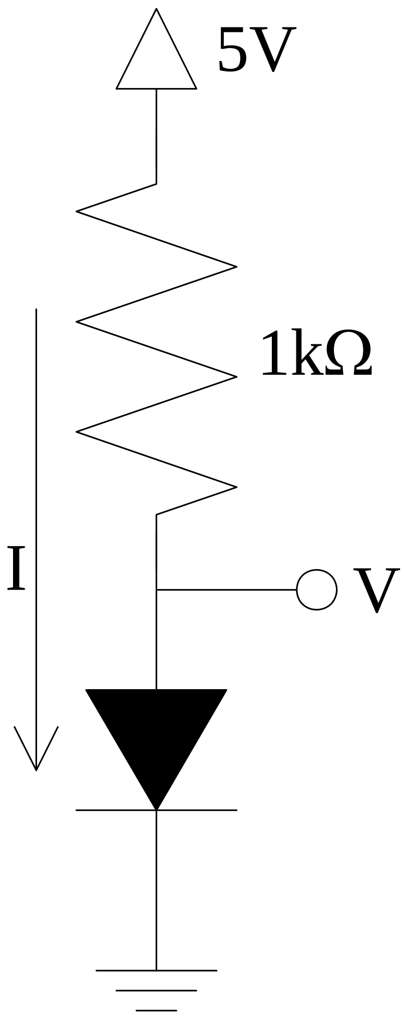
Reverse Bias

- Recalling exponential model $$i = I_S\left(e^{\frac{v}{nV_T}} - 1\right)$$
- As *v* becomes negative,

$$e^{\left(\frac{v}{nV_T}\right)} << 1 \qquad i = -I_S$$

- Exponential model predicts approximately constant current under reverse bias; IC Si devices: $I_S \sim 10^{-15}$
- Usually, consider a reverse-biased diode to be *nonconductive; open circuit*

Circuit Analysis



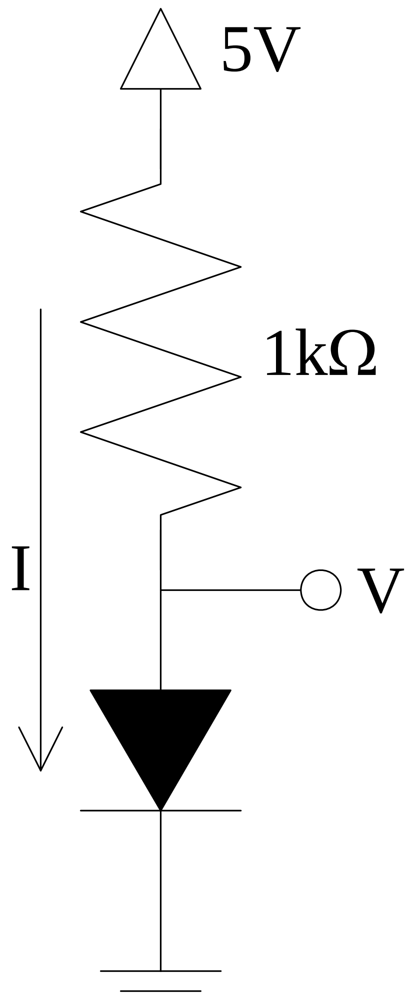
Given: n = 1, $I_S = 6.9\text{x}10^{-16}$A
Find: I and V

- For resistor: $$I = \frac{5 - V}{1k}$$
- For diode: $$I = I_S \exp\left(\frac{V}{n \cdot V_T}\right)$$

$$I = \frac{5 - V}{1k} = 6.9 \times 10^{-16} \exp\left(\frac{V}{25mV}\right)$$

- This is generally best for a circuit simulator to solve (like SPICE)

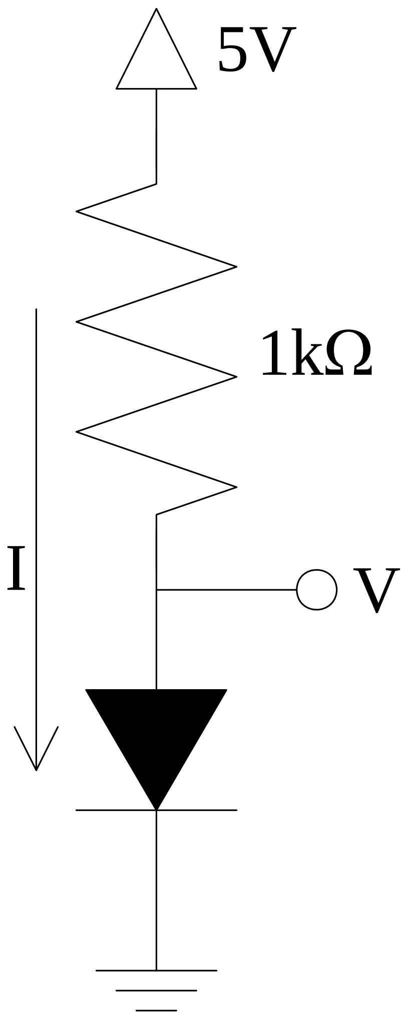
Iteration



$n = 1$, $I_S = 6.9\times10^{-16}$A **Find I and V**

- Iterative analysis procedure:
  - Start with a guess for diode voltage drop — V ≈ 0.7 is reasonable
  - Use guess for V to get corresponding I — $I = \frac{5 - V}{1k}$
  - Use I to get better approximation for V — $V = (25mV)\ln\left(\frac{I}{6.9\times10^{-16}}\right)$
  - Repeat procedure until V and I no longer change

Iteration (cont')



n = 1, $I_S = 6.9 \times 10^{-16}$A **Find I and V**

- Iteration #1 (V = 0.7V)

$$I = \frac{5 - 0.700}{1k} = 4.300mA$$

$$V = (25m)\ln\left(\frac{4.300m}{6.9 \times 10^{-16}}\right) = 0.737V$$

- Iteration #2 (V = 0.737V)

$$I = \frac{5 - 0.737}{1k} = 4.263mA$$

$$V = (25m)\ln\left(\frac{4.263m}{6.9 \times 10^{-16}}\right) = 0.736V$$

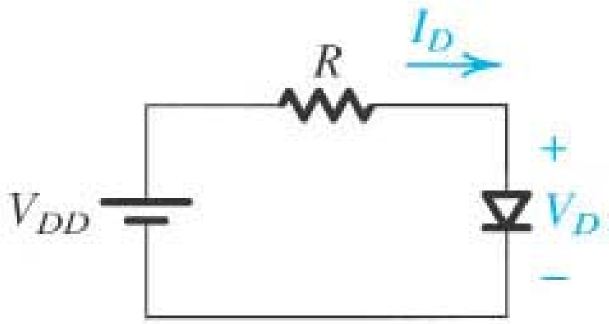
- Iteration #3 (V = 0.736V)

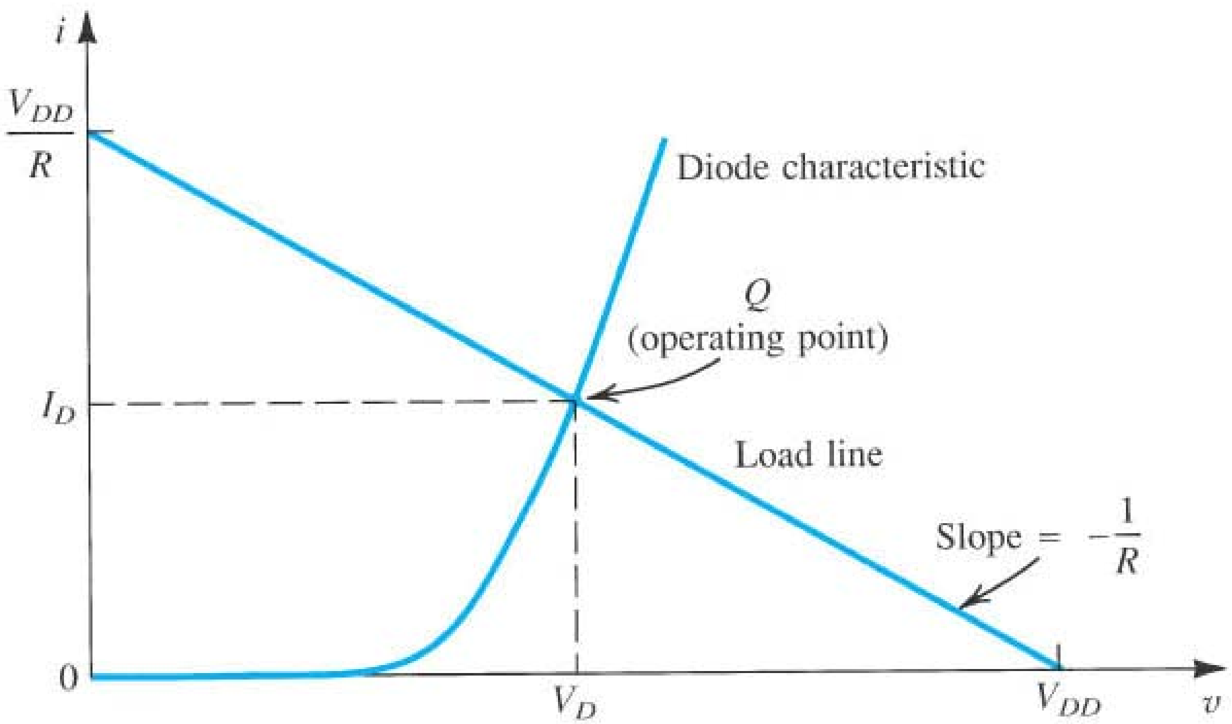
$$I = \frac{5 - 0.736}{1k} = 4.264mA$$

$$V = (25m)\ln\left(\frac{4.264m}{6.9 \times 10^{-16}}\right) = 0.736V$$

∴ I = 4.264mA, V = 0.736V

Graphical Analysis





1) Plot two relationships on the i-v plane.

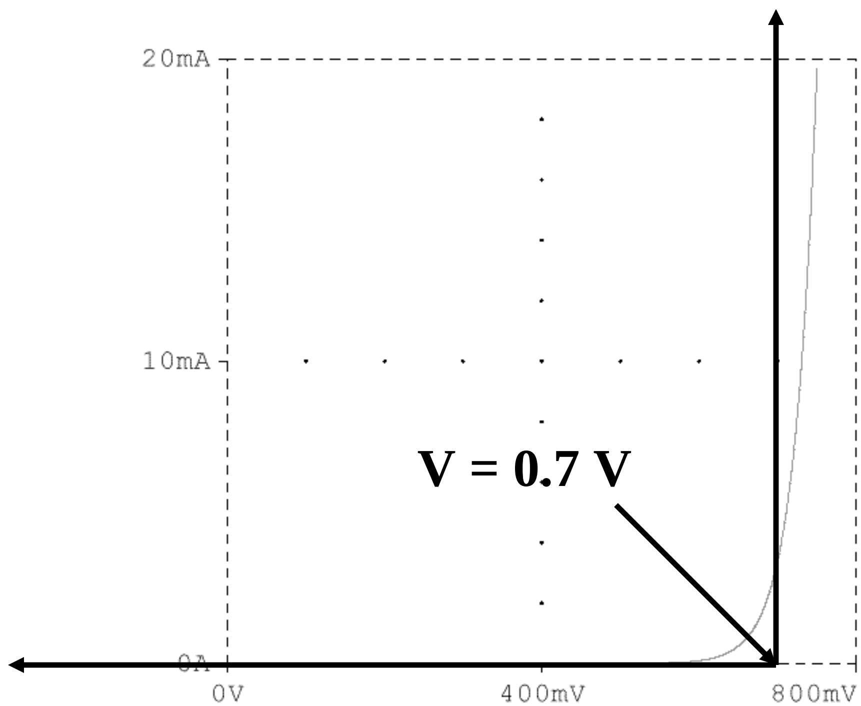
2) The solution is the intersection of the two graphs; ***operating point***

Outline of Section 3 - Diodes

- Other two terminal devices
- Diode models
- Exponential model
- *<u>Constant voltage drop model</u>*
- Reverse breakdown
- Applications
- Small-signal model
- PN junctions

The Constant Voltage Drop Model (CVDM)

- Exponential model gives accurate results; requires hand computation or a simulator
- The constant voltage drop model (CVDM) used to perform quick analysis of a diode circuit by hand
- CVDM approximates diode I-V curve piecewise-linearly

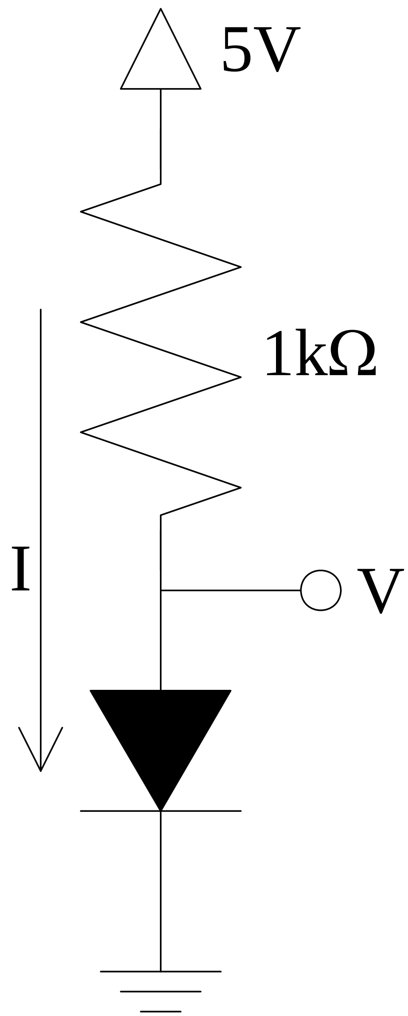


CVDM

- "Voltage" perspective of CVDM:
  - V = 0.7V when diode is conducting
  - V < 0.7V ⇨ diode is not conducting

- "Current" perspective of CVDM:
  - When V = 0.7V, diode supplies whatever current is required by the circuit
  - When V < 0.7V, diode supplies no current

How to use CVDM to Find I and V



1) Make assumptions about whether diodes are *conducting or not*

2) Solve circuit:

   use *0.7V drops* for conducting diodes

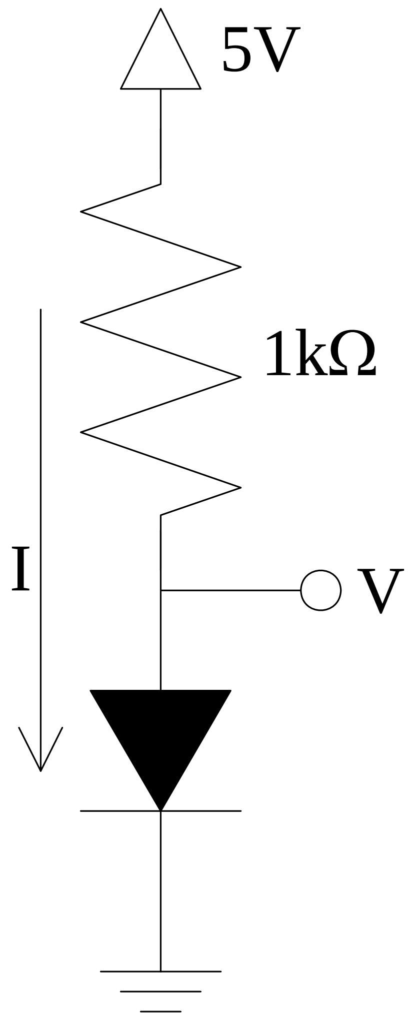
   treat non-conducting diodes as *open-circuits*

3) Check validity of assumptions:

   If consistent ⇨ DONE

   If inconsistent ⇨ repeat with new assumptions

Example 1 – Find I and V



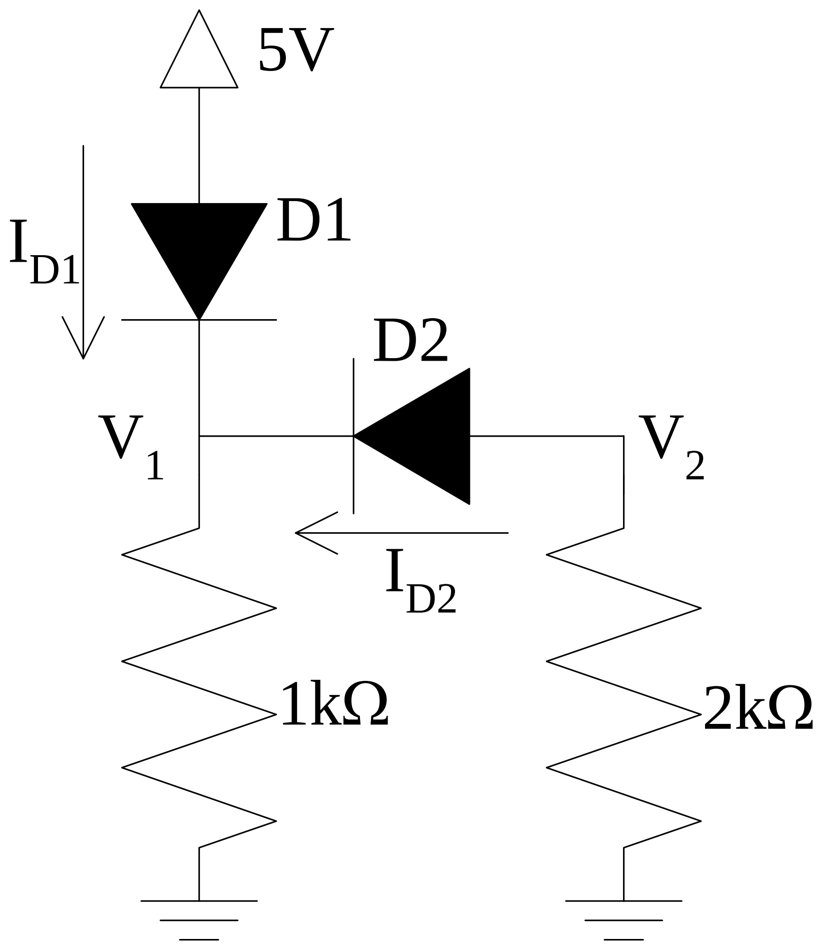
Assume that diode is conducting:

$$\boxed{V = 0.7V}$$

$$\boxed{I = \frac{5-0.7}{1k} = 4.3mA}$$

Result indicates that diode voltage drop 0.7V and diode current is 4.3 mA – acceptable.

Example 2 - Find V_1, V_2, I_{D1} and I_{D2}



Assume that both diodes are conducting

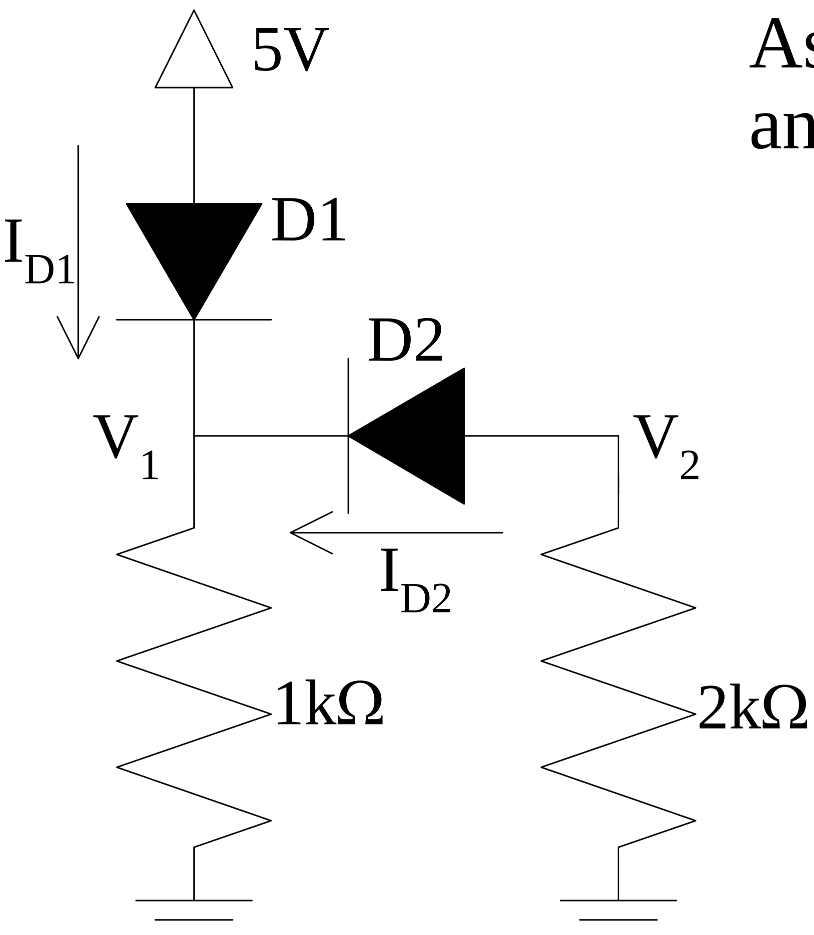
$$V_1 = 5 - 0.7 = \underline{4.3V}$$

$$V_2 - V_1 = 0.7 \Rightarrow \underline{V_2 = 5V}$$

$$I_{D2} = \frac{0 - V_2}{2k} = \frac{-5}{2k} = \underline{-2.5mA}$$

Results ***not*** consistent for D_2

New Assumptions



Assume that D_1 is conducting and that D_2 is not conducting

$$V_1 = 5 - 0.7 = \underline{4.3V}$$

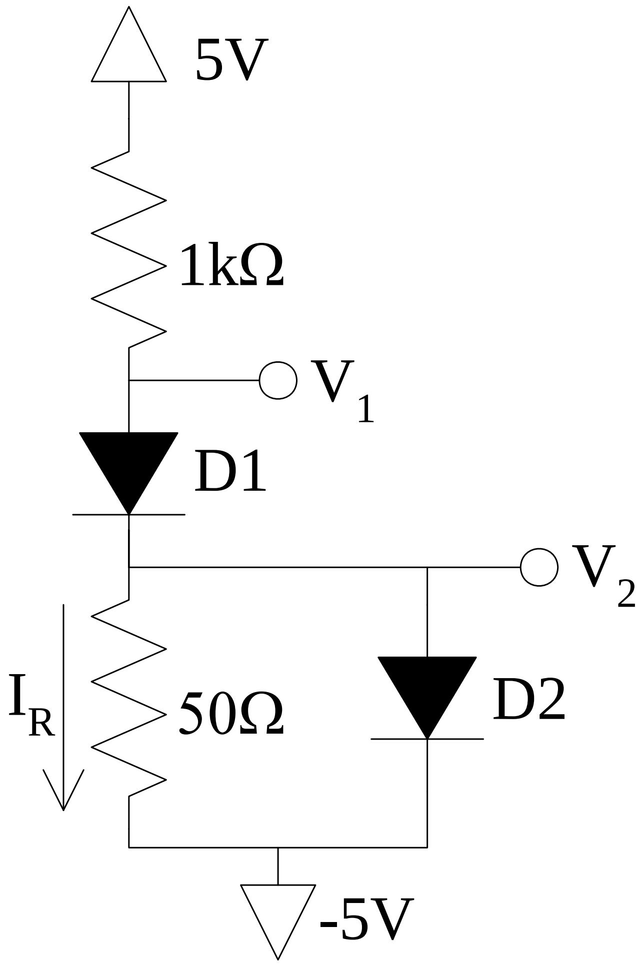
$$\underline{I_{D2} = 0A}$$

$$\underline{V_2 = 0V}$$

$$I_{D1} = \frac{V_1 - 0}{1k} = \underline{4.3mA}$$

Results are acceptable

Example 3 - Find V_1, V_2, I_{D1} and I_{D2}



Assume both D1 and D2 conducting

$$V_2 = -5 + 0.7 = \underline{-4.3V}$$

$$V_1 = V_2 + 0.7 = \underline{-3.6V}$$

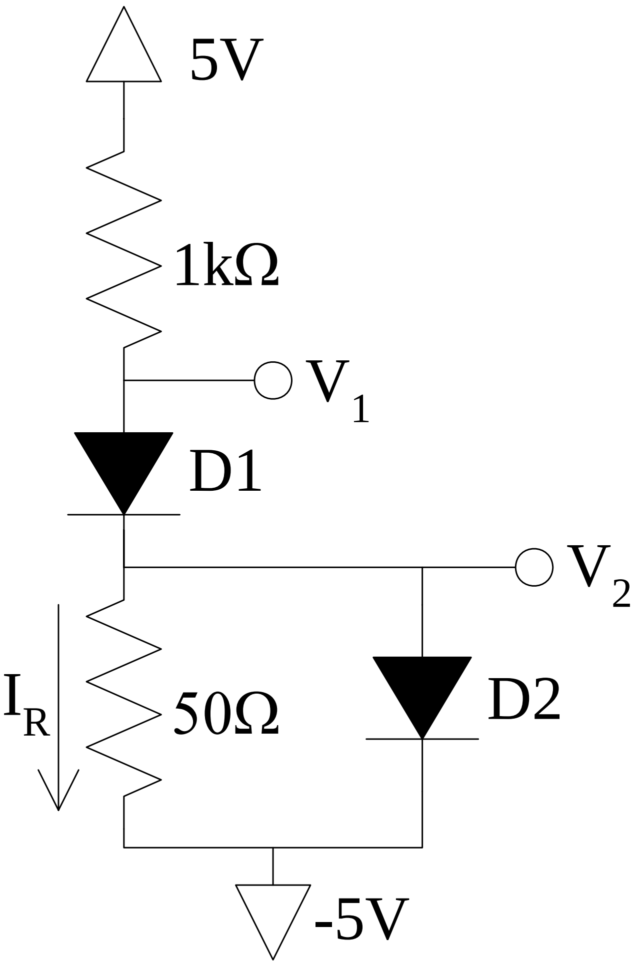
$$I_{D1} = \frac{5 - V_1}{1k} = \frac{5 + 3.6}{1k} = \underline{8.6mA}$$

$$I_R = \frac{0.7}{50} = \underline{14mA}$$

$$I_{D2} = I_{D1} - I_R = \underline{-5.4mA}$$

Results ***not*** consistent for D2

New Assumptions



Assume D1 on, D2 off

$$I_{D1} = I_R = \frac{5 - V_1}{1k} = \frac{V_2 + 5}{50}$$

$$V_1 - V_2 = 0.7V$$

$$\frac{5 - (V_2 + 0.7)}{1k} = \frac{V_2 + 5}{50} \Rightarrow V_2 = \underline{-4.557V}$$

$$V_1 = \underline{-3.857V}$$

$$I_{D1} = \frac{5 - V_1}{1k} = \underline{8.857mA}$$

Check D2: $V_2 + 5 = 0.443V < 0.7V \rightarrow D2\ off$

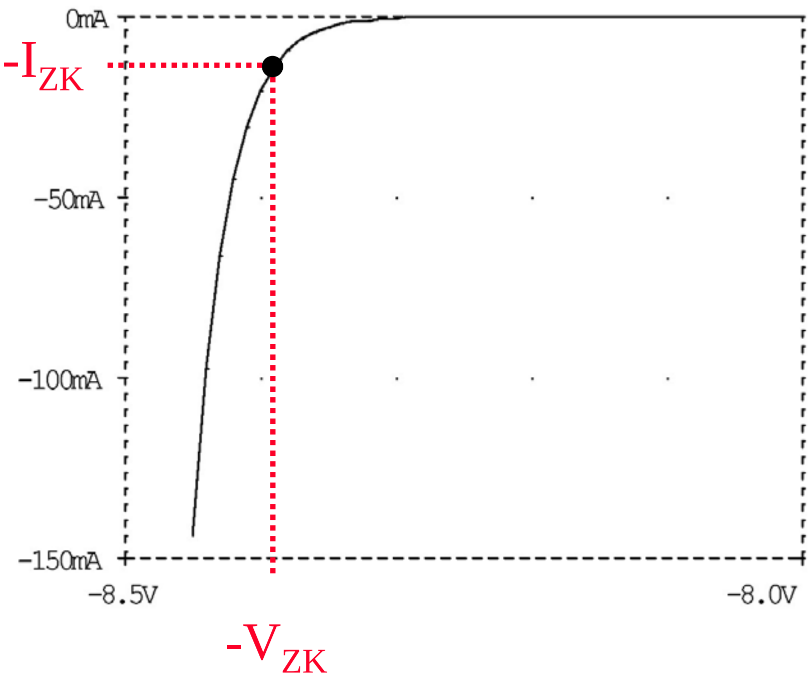
Results acceptable

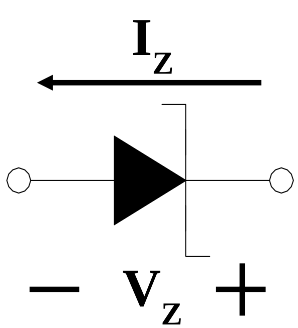
Outline of Section 3 - Diodes

- Other two terminal devices
- Diode models
- Exponential model
- Constant voltage drop model
- *Reverse breakdown*
- Applications
- Small-signal model
- PN junctions

Reverse-Breakdown Region – Characteristics

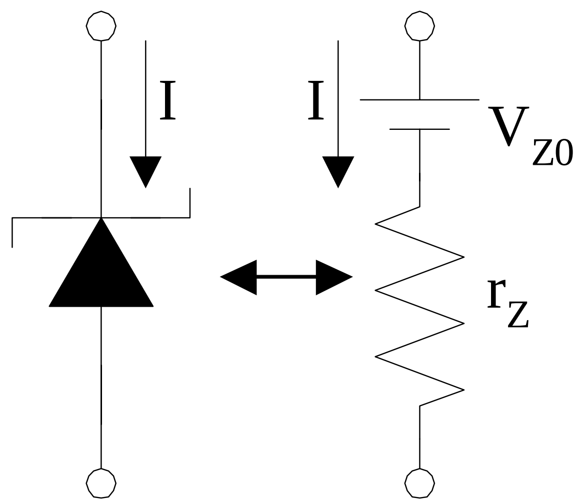
- Point on I-V curve where breakdown occurs called ***Zener*** knee ($-V_{ZK}$, $-I_{ZK}$)
- ***Zener*** diodes designed specifically for operation in reverse-breakdown.
- This means that they can handle large currents, hence they are physically larger.





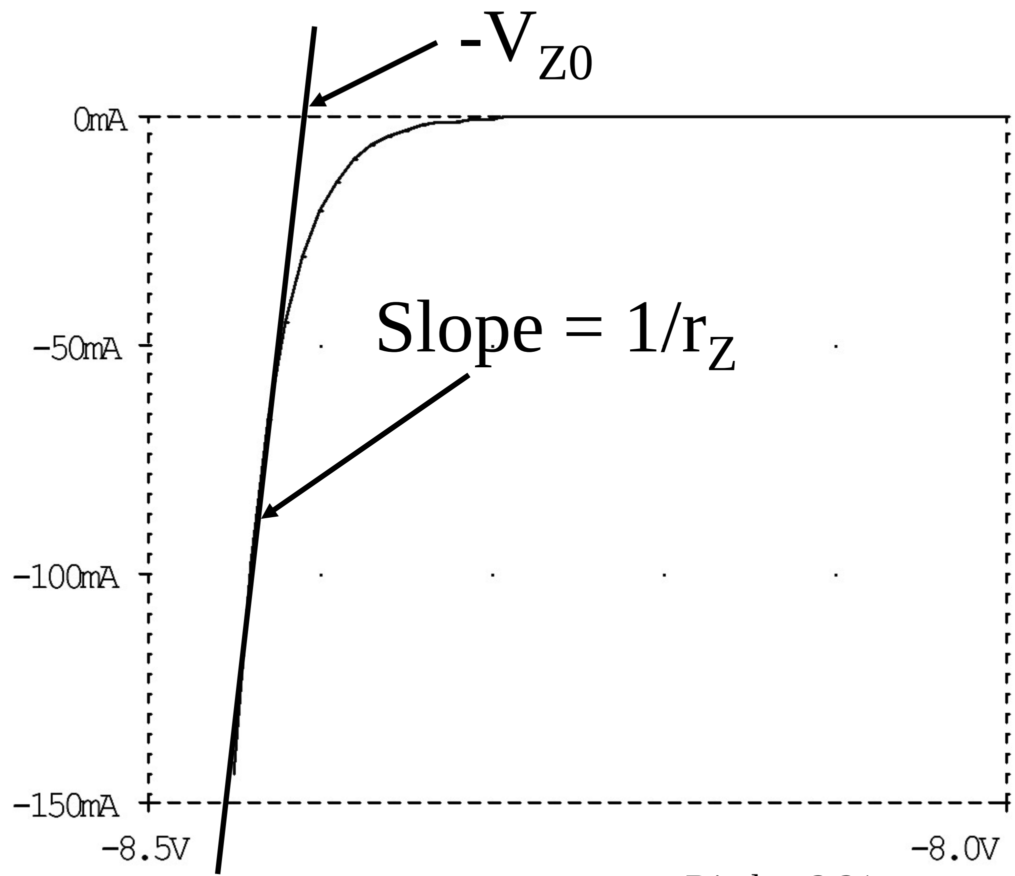
Reverse-Breakdown Region – Modeling

- Model for Zener diode:



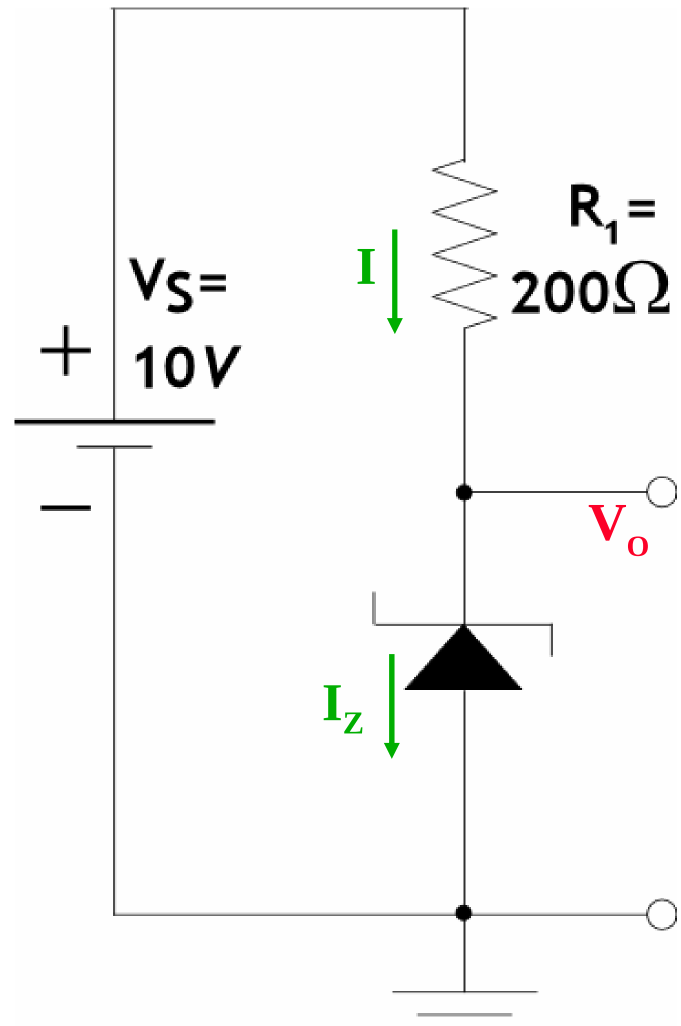
- Slope of I-V curve in reverse-breakdown region very steep; r_Z very small

- Typical Zener application : voltage regulation



Zener Example

Question: given a Zener Diode with V_{Z0} = 5.5V and an incremental resistance of r_z=40Ω, calculate the output voltage V_O.



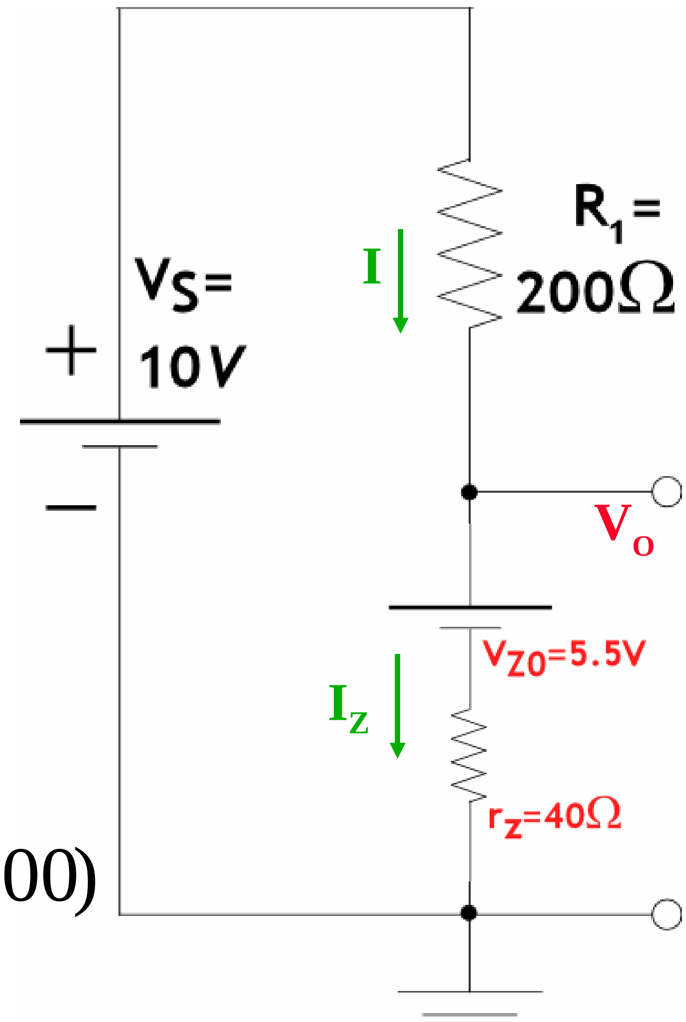
Example (cont')

1) Replace Zener with model
2) Perform circuit analysis by solving for I in the network:

$$V_S = IR_1 + V_{Z0} + Ir_Z$$

$$10 = I(200) + 5.5 + I(40)$$

$$\Rightarrow I = 18.75mA$$



3) Compute V_o:

$$V_O = V_S - IR_1 = 10 - (18.75mA)(200)$$

$$\Rightarrow V_O = 6.25V$$