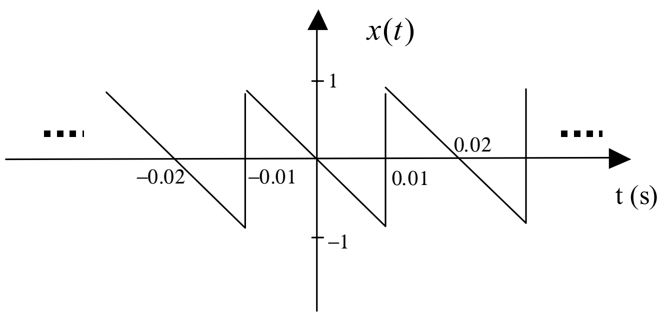
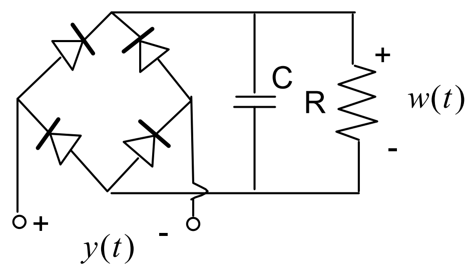
Sample Midterm Test 2 (mt2s02)
Covering Chapters 13-16 of *Fundamentals of Signals & Systems*

Problem 1 (30 marks)

You have to design the envelope detector shown below to demodulate asynchronously an AM signal $y(t)$ with modulation index $m = 0.5$ and carrier frequency $f_c = 1\,\mathrm{kHz}$, where $x(t)$ is the periodic modulating signal shown below. The envelope detector is implemented with a simple RC circuit at the output of a full-wave diode bridge rectifier.

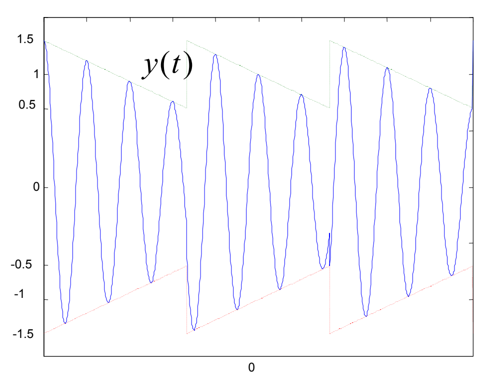




(a) *[5 marks]* Assuming that the carrier has to be transmitted with unity amplitude, write the expression for the signal $y(t)$. Provide a rough sketch of $y(t)$.

Answer:

$$y(t) = [1 + 0.5x(t)]\cos(2\pi 10^3 t)$$



(b) *[25 marks]* Compute the values of the circuit components. Justify all of your approximations and assumptions. Provide a rough sketch of the signal at the output of the detector.

Answer:

The output voltage of the detector, when it goes from one peak at voltage v_1 to the next when it intersects the modulated carrier at voltage v_2 after approximately half a period $T_0 = \frac{T}{2} = 0.5ms$ of the carrier, is given by:

$$v_2 \cong v_1 e^{-T_0/RC}$$

Since the time constant $\tau = RC$ of the detector should be large with respect to $T_0 = 0.5ms$, we can use a first-order approximation of the exponential such that

$$v_2 \cong v_1(1 - T_0/RC).$$

This is a line of negative slope $-\frac{v_1}{RC}$ between the initial voltage v_1 and the final voltage v_2 so that

$$\frac{v_2 - v_1}{T_0} \cong -v_1/RC.$$

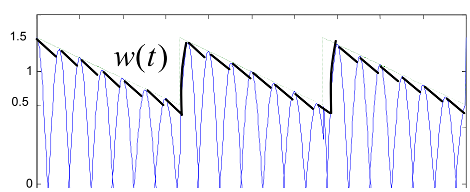
This slope must be more negative than the maximum negative slope of the envelope of $y(t)$, which is maximized as follows:

$$\min_t \frac{d(0.5x(t))}{dt} = -50$$

Taking the worst-case $v_1 = 0.5$, we must have

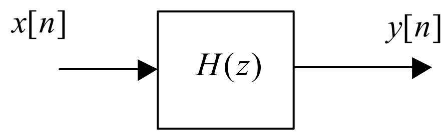
$$-\frac{0.5}{RC} < -50 \Leftrightarrow RC < \frac{0.5}{50} = 0.01$$

We could take $R = 5k\Omega,\ C = 1\mu F$ to get $RC = 0.005$.



Problem 2 (30 marks)

Consider the DLTI system



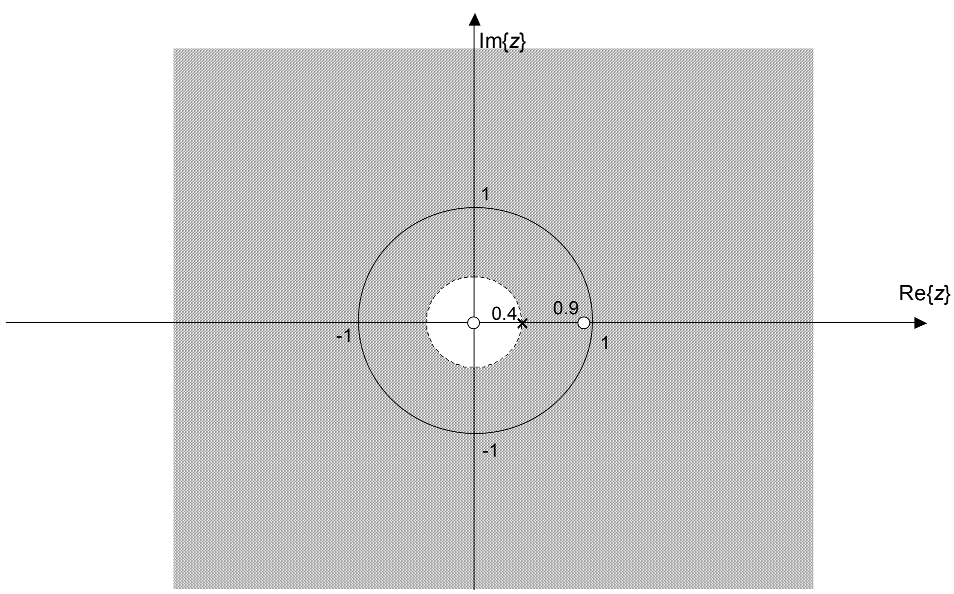
with transfer function $H(z) = \dfrac{1-0.9z^{-1}}{z^{-2}(z-0.4)}$.

(a) [8 marks] Sketch the pole-zero plot of the system.

Answer:

$$H(z) = \frac{z^2 - 0.9z}{(z-0.4)} = \frac{z(z-0.9)}{z-0.4}$$

pole at 0.4 and infinity, zeros at 0 and 0.9.



(b) [5 marks] Find the ROC that makes this system stable.

Answer:

For stability, the ROC must include the unit circle, so it is $|z| > 0.4$, excluding infinity.

(c) [5 marks] Is the system causal with the ROC that you found in (b)? Justify your answer.

Answer:

For the system to be causal, we must have $\lim_{z\to\infty} H(z) < \infty$, which is not the case here. Hence the system is not causal.

(d) [12 marks] Suppose that $H(e^{j\omega})$ is bounded for all frequencies. Calculate the response of the system $y[n]$ to the input $x[n]=u[n]$.

Answer:

This means that the ROC is chosen as $|z|>0.4$.

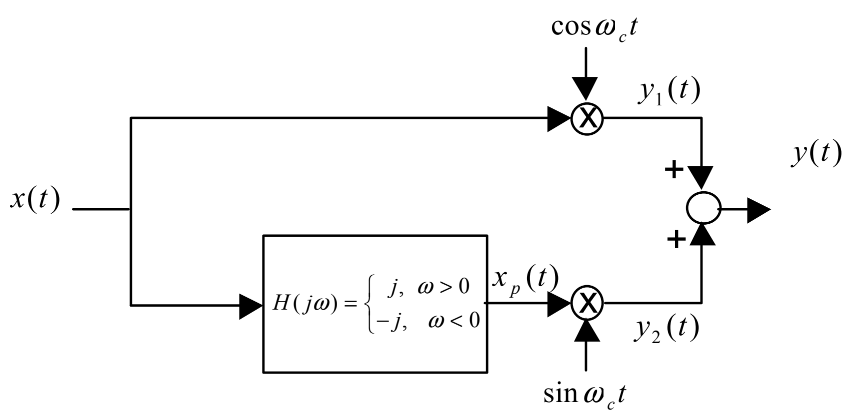
We have $X(z)=\dfrac{1}{1-z^{-1}}$, $|z|>1$, and

$$Y(z)=H(z)U(z)=\frac{z^2(z-0.9)}{(z-0.4)(z-1)}=\frac{z(1-0.9z^{-1})}{(1-0.4z^{-1})(1-z^{-1})}=z\left[\underbrace{\frac{5/6}{1-0.4z^{-1}}}_{|z|>0.4}+\underbrace{\frac{1/6}{1-z^{-1}}}_{|z|>1}\right]$$

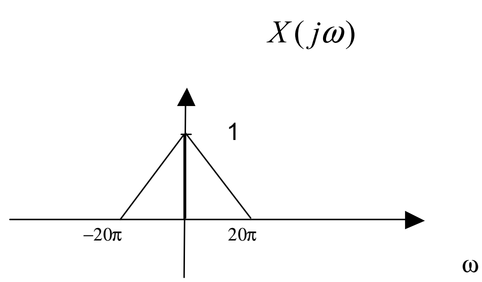
so $y[n]=\dfrac{5}{6}(-0.4)^{n+1}u[n+1]+\dfrac{1}{6}u[n+1]$

Problem 3 (40 marks)

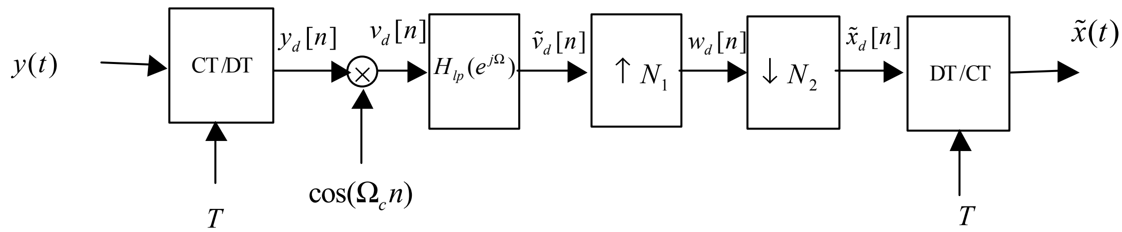
The SSB modulator shown below produces a signal that must be demodulated in discrete-time.



The modulating signal has a Fourier transform as shown below, and the carrier frequency is $\omega_c=2\,000\pi$ rd/s.



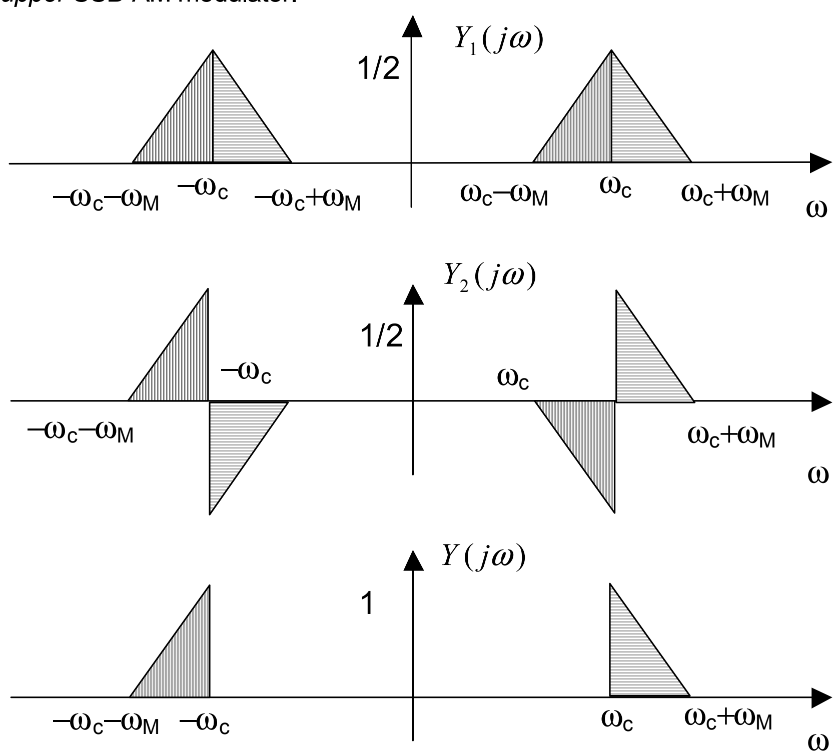
Suppose the system is entirely noise-free so that an antialiasing filter is not required. The discrete-time demodulator is shown below. The ideal lowpass filter has unity gain and cutoff Ω_{lp}.



(a) *[10 marks]* Sketch the spectra $Y_1(j\omega)$, $Y_2(j\omega)$ and $Y(j\omega)$ of the corresponding signals in the SSB modulator.

Answer:

This is an *upper* SSB-AM modulator.

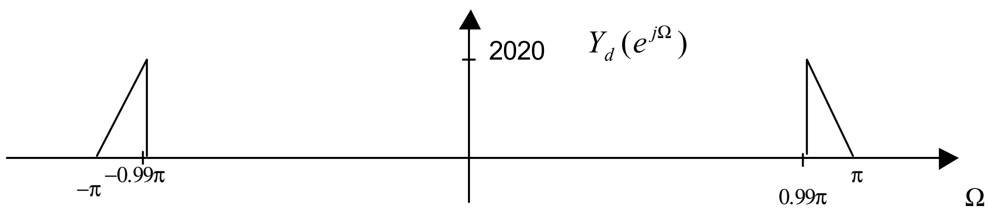


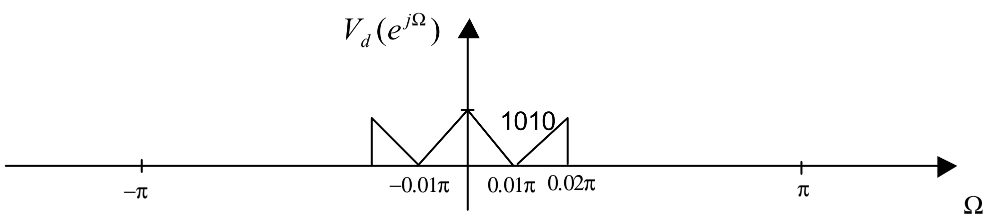
where $\omega_c = 2000\pi, \omega_M = 20\pi$.

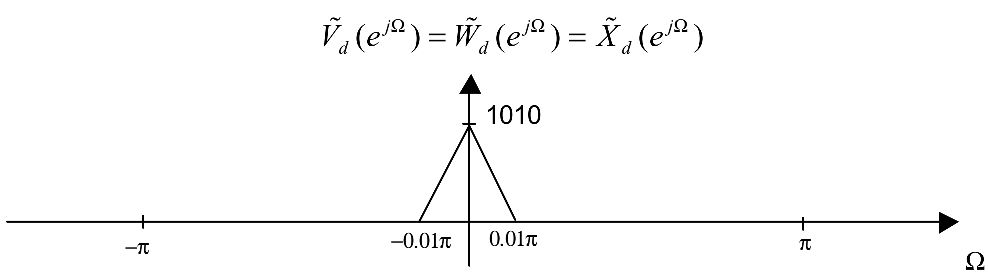
(b) *[20 marks]* Assuming that $N_1 = N_2 = 1$ (no decimation/upsampling), find the minimum sampling frequency ω_s that satisfies the sampling theorem with respect to the bandwidth of the modulated signal and that will allow perfect recovery of the modulating signal $\tilde{x}(t) = x(t)$. Give the corresponding DT demodulation frequency Ω_c and filter cutoff frequency Ω_{lp}. Sketch the spectra $Y_d(e^{j\Omega})$, $V_d(e^{j\Omega})$ and $\tilde{X}_d(e^{j\Omega})$ of the corresponding signals in the demodulator for the sampling frequency that you found. Indicate the important frequencies and magnitudes on your sketches.

Answer:

Minimum $\omega_s = 2(\omega_c + \omega_m) = 4040\pi$. Thus the sampling period is
$T = \dfrac{2\pi}{4040\pi} = \dfrac{1}{2020} = 0.000495$. Demodulation frequency is therefore
$\Omega_c = T\omega_c = \dfrac{2000\pi}{2020} = 0.9901\pi$ and the filter cutoff should be at $\Omega_{lp} = 0.01\pi$.







(c) *[10 marks]* Using the same parameters as in (b), except for $N_1 = 4, N_2 = 2$, sketch the spectra $W_d(e^{j\Omega})$, $\tilde{X}_d(e^{j\Omega})$ and $\tilde{X}(j\omega)$.

Answer: