Solutions to Problems in Chapter 11

Problems with Solutions

Problem 11.1

Consider the causal LTI unity feedback regulator in Figure 11.1, where

$$P(s) = \frac{(s-1)}{(s+1)(s^2+5\sqrt{2}s+25)},\quad K(s) = \frac{s^2+3\sqrt{2}s+9}{(s+2)(s+3)} \text{ and } k \in [0, +\infty).$$

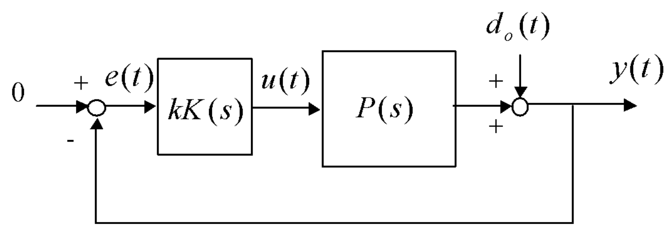


Figure 11.1: Regulator of root locus Problem 11.1.

(a) Use properties of the root locus to sketch it. Check your sketch using Matlab (*rlocus* command).

Answer:

The loop gain is $L(s) = kP(s)K(s) = \dfrac{k(s-1)(s^2+3\sqrt{2}s+9)}{(s+1)(s+2)(s+3)(s^2+5\sqrt{2}s+25)}$.

- The root locus starts at the (open-loop) poles of $L(s)$: $-1, -2, -3, -\frac{5}{\sqrt{2}} \pm \frac{5}{\sqrt{2}} j$ for $k=0$ and it ends at the zeros of $L(s)$: $1, -\frac{3}{\sqrt{2}} \pm \frac{3}{\sqrt{2}} j, \infty, \infty$ for $k = +\infty$.

- On the real line, the root locus will have one branch between the pole at -1 and the zero at -2, and also one branch between the poles at -3 and -2 (Rule 4)

- Let $\nu = \deg\{p_L\} = 5$ and $\mu = \deg\{n_L\} = 3$. For the two branches of the root locus going to infinity, the asymptotes are described by

$$\text{Center of asymptotes} \quad \begin{aligned} &= \frac{\sum \text{poles of } L(s) - \sum \text{zeros of } L(s)}{\nu - \mu} \\ &= \frac{(-1-2-3-5\sqrt{2}) - (1-3\sqrt{2})}{2} = \frac{-7-2\sqrt{2}}{2} = -4.9 \end{aligned}$$

$$\text{Angles of asymptotes} = \frac{2k+1}{\nu-\mu}\pi, \quad k = 0,1 \quad \Rightarrow \quad = \begin{cases} \pi/2, & k = 0 \\ 3\pi/2, & k = 1 \end{cases}$$

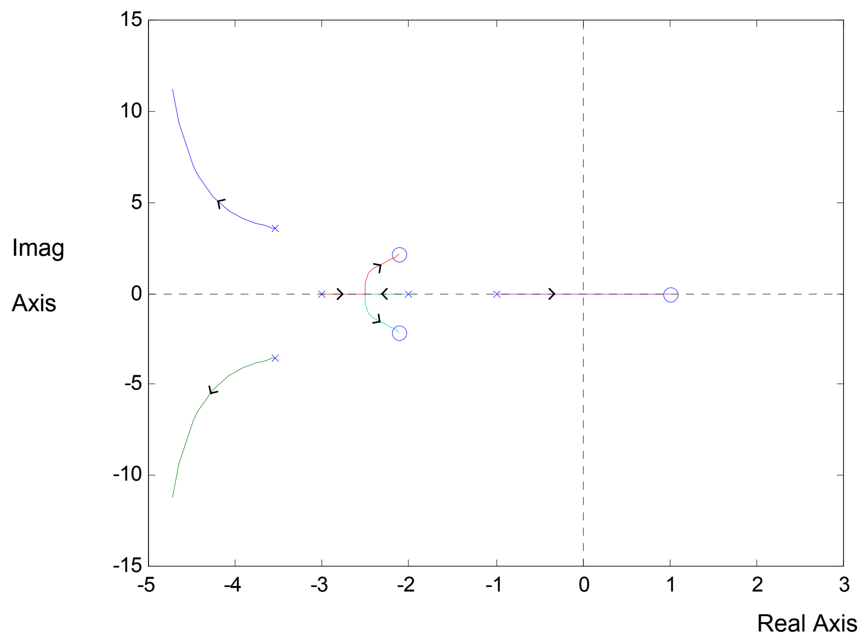


Figure 11.2: Root locus Problem 11.1.

(b) Compute the value of the controller gain k for which the control system becomes unstable.

Answer:

The characteristic polynomial is obtained:

$$\begin{aligned} p(s) &= n_L(s) + d_L(s) \\ &= k(s-1)(s^2+3\sqrt{2}s+9)+(s+1)(s+2)(s+3)(s^2+5\sqrt{2}s+25) \\ &= s^5+a_4s^4+a_3s^3+a_2s^2+a_1s+(-9k+150) \end{aligned}$$

From the root locus, we see that the onset of instability occurs when a real pole crosses the imaginary axis at 0. At that point, we have:

$$\begin{aligned} & p(s)\big|_{s=0} = -9k+150 = 0 \\ & \Rightarrow k = \frac{50}{3} = 16.67 \end{aligned}$$

Problem 11.2

We wish to analyze the stability of the tracking control system of , where $P(s) = \dfrac{4s+1}{(s-1)(0.1s+1)}$ and $K(s) = \dfrac{10(0.5s+1)}{0.1s+1}$.

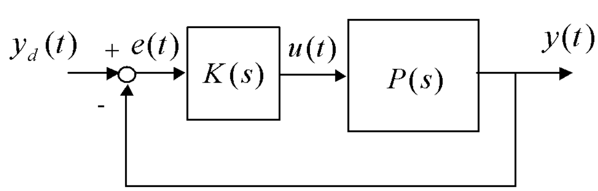


Figure 11.3: Tracking system of Problem 11.2.

(a) Assess the stability of this closed-loop system using any two of the four stability theorems.

Using Theorem I:

We have to show that $T(s)$, $P^{-1}(s)T(s)$ and $P(s)S(s)$ are all stable.

$$T(s) = \frac{P(s)K(s)}{1+P(s)K(s)} = \frac{\dfrac{10(4s+1)(0.5s+1)}{(s-1)(0.1s+1)^2}}{1+\dfrac{10(4s+1)(0.5s+1)}{(s-1)(0.1s+1)^2}} = \frac{20s^2+45s+10}{0.01s^3+20.19s^2+45.8s+9}$$

is stable (all three poles in LHP, strictly proper)

poles are: -2017, -2.1, -0.2

$$P^{-1}(s)T(s) = \frac{\dfrac{10(0.5s+1)}{0.1s+1}}{1+\dfrac{10(4s+1)(0.5s+1)}{(s-1)(0.1s+1)^2}} = \frac{(0.1s^2+0.9s-1)(5s+10)}{0.01s^3+20.19s^2+45.8s+9}$$

is also stable (same poles as above, proper)

$$P(s)S(s) = \frac{P(s)}{1+P(s)K(s)} = \frac{\dfrac{4s+1}{(s-1)(0.1s+1)}}{1+\dfrac{10(4s+1)(0.5s+1)}{(s-1)(0.1s+1)^2}} = \frac{0.4s^2+4.1s+1}{0.01s^3+20.19s^2+45.8s+9}$$

is stable (same poles as above, strictly proper)

Therefore the feedback system is stable.

Using Theorem II:

We have to show that either $T(s)$ or $S(s)$ is stable, and that no pole-zero cancellation occurs in the closed RHP in forming the loop gain. The latter condition holds, but

$$T(s) = \frac{P(s)K(s)}{1+P(s)K(s)} = \frac{\dfrac{10(4s+1)(0.5s+1)}{(s-1)(0.1s+1)^2}}{1+\dfrac{10(4s+1)(0.5s+1)}{(s-1)(0.1s+1)^2}} = \frac{20s^2+45s+10}{0.01s^3+20.19s^2+45.8s+9}$$

is stable (all three poles in LHP, strictly proper)

poles are: -2017, -2.1, -0.2

Therefore the feedback system is stable.

Using Theorem III:

We have to show that the closed-loop poles, i.e., the zeros of the characteristic polynomial $p(s)$, are all in the open LHP. The plant and the controller are already expressed as ratios of coprime polynomials.

$$p(s) = n_K n_P + d_K d_P = 0.01s^3 + 20.19s^2 + 45.8s + 9$$

All three closed-loop poles -2017, -2.1, -0.2 lie in the open LHP and therefore the feedback system is stable.

Using Theorem IV:

We have to show that $1+K(s)P(s)$ has no zero in the closed RHP, and that no pole-zero cancellation occurs in the closed RHP in forming the loop gain. The latter condition obviously holds, and

$$1+K(s)P(s) = \frac{1}{S(s)} = 1+\frac{10(4s+1)(0.5s+1)}{(s-1)(0.1s+1)^2} = \frac{0.01s^3+20.19s^2+45.8s+9}{0.01s^3+0.19s^2+0.8s-1}$$

All three zeros of this TF -2017, -2.1, -0.2 lie in the open LHP and therefore the feedback system is stable.

(b) Use Matlab to sketch the Nyquist plot of the loop gain $L(s) = K(s)P(s)$, give the critical point and discuss the stability of the closed-loop system using the Nyquist criterion.

Answer:

The Nyquist plot, given in Figure 11.4 was produced using the 'nyquist' command in Matlab.

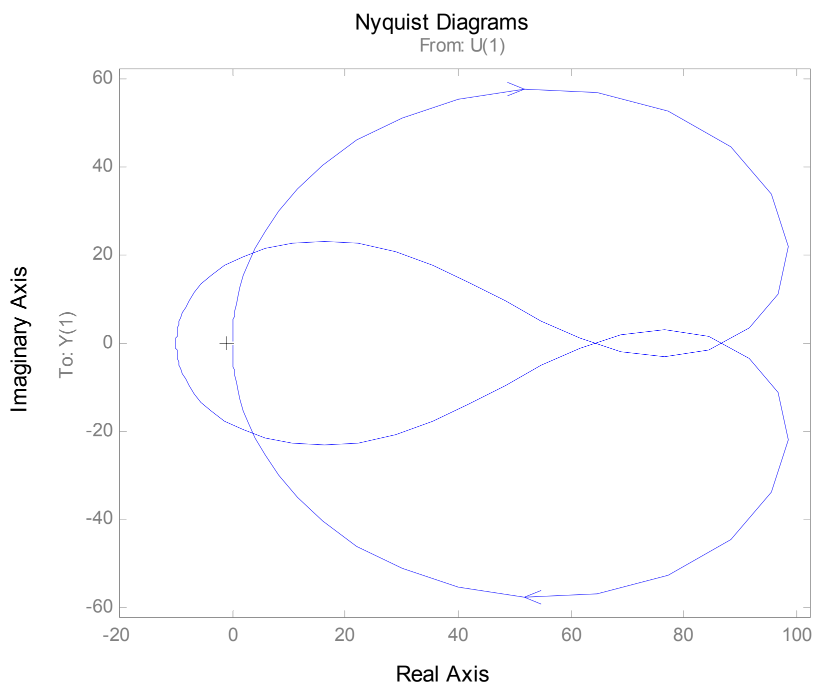


Figure 11.4: Nyquist plot of Problem 11.2(b).

The loop gain has one RHP pole and according to the Nyquist criterion, the Nyquist plot should encircle the critical point -1 once counterclockwise. This is indeed what we can observe here, and hence the closed-loop system is stable.

Problem 11.3

We wish to analyze the stability of the tracking control system in Figure 11.5, where $P(s)=\dfrac{s+1}{10s+1}$ and $K(s)=\dfrac{10}{s}$.

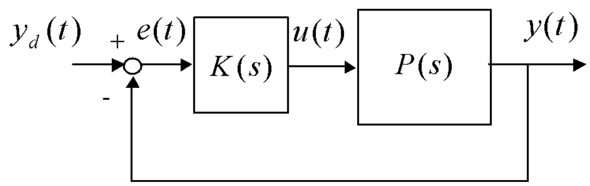


Figure 11.5: Tracking control system of Problem 11.3.

Sketch the Bode plot in dBs (magnitude) and degrees (phase) of the loop gain (you can use Matlab to check if your sketch is right). Find the frequencies ω_{co} and $\omega_{-\pi}$. Assess the stability robustness by finding the gain margin k_m and phase margin ϕ_m of the system. Compute the minimum time delay in the plant that would cause instability.

Answer:

The loop gain is $L(s)=\dfrac{10(s+1)}{s(10s+1)}$ and its Bode plot is shown in Figure 11.6.

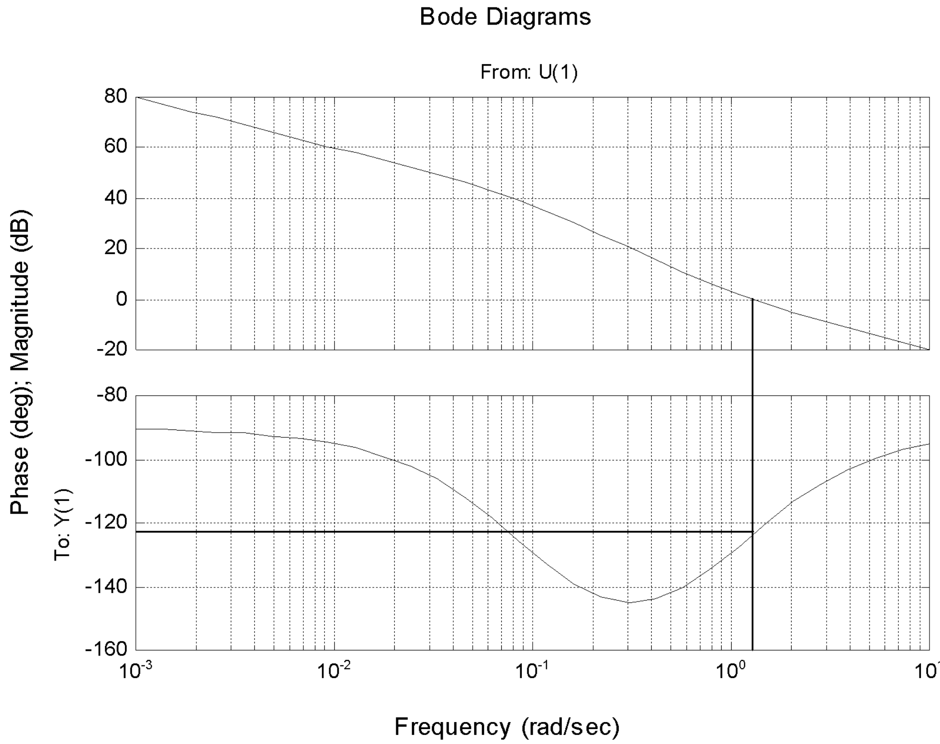


Figure 11.6: Bode plot of loop gain in Problem 11.3.

Frequency $\omega_{-\pi}$ is undefined as the phase never reaches –180deg. Thus, the gain margin k_m is infinite, and the phase margin is $\phi_m \cong 58^\circ$ at $\omega_{co} \cong 1.3\text{rd/s}$. The minimum time delay τ in the plant that would cause instability is obtained as follows:

$$\omega_{co}\tau = \frac{\pi}{180}\phi_m$$
$$\Rightarrow \tau = \frac{\pi}{180\omega_{co}}\phi_m = \frac{58\pi}{180 \cdot 1.3} = 0.78\,\text{s}$$

Exercises

Problem 11.4

Consider the LTI feedback control system shown in Figure 11.7.

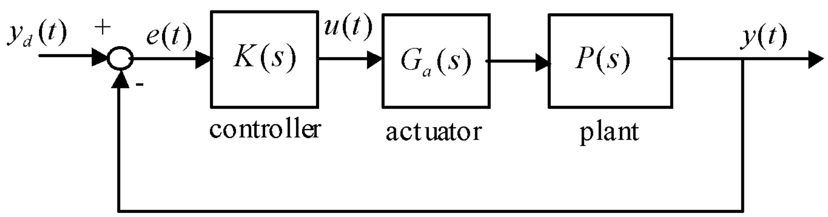


Figure 11.7: Feedback control system of Problem 11.4.

The transfer functions of the different causal components of the control system are $G_a(s) = \dfrac{1}{s+1}$, $K(s) = \dfrac{4s+8}{s+10}$ and $P(s) = \dfrac{s-1}{10s+1}$.

(a) Compute the loop gain $L(s)$, and the closed-loop characteristic polynomial $p(s)$. Is the closed-loop system stable?

(b) Find the sensitivity $S(s)$ and complementary sensitivity $T(s)$ functions.

(c) Find the steady-state error signal of the closed-loop response $y(t)$ to the input $y_d(t) = \sin(t)$.

(d) Assume that the real parameter k is an additional gain on the controller, i.e., the controller is $kK(s)$. Sketch the root locus for the real parameter varying in $k \in [0, +\infty)$.

Problem 11.5

We wish to analyze the stability of the tracking control system shown in Figure 11.8, where $P(s) = \dfrac{-0.1s+1}{(0.01s+1)(0.1s+1)}$ and $K(s) = \dfrac{10(0.5s+1)}{s}$. Assess the stability of this closed-loop system using any two of the four stability theorems.

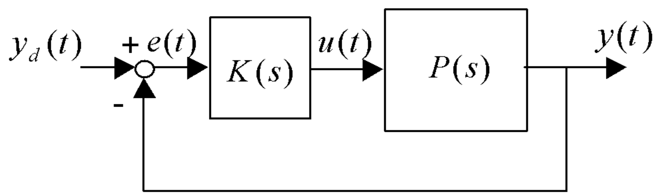


Figure 11.8: Feedback control system of Problem 11.8.

Answer:

Using Theorem I:

We have to show that $T(s)$, $P^{-1}(s)T(s)$ and $P(s)S(s)$ are all stable.

$$T(s) = \frac{P(s)K(s)}{1+P(s)K(s)} = \frac{\dfrac{10(0.5s+1)(-0.1s+1)}{s(0.01s+1)(0.1s+1)}}{1+\dfrac{10(0.5s+1)(-0.1s+1)}{s(0.01s+1)(0.1s+1)}} = \frac{-0.5s^2+4s+10}{0.001s^3-0.39s^2+5s+10}$$

$$= \frac{-500(s^2-8s-20)}{s^3-390s^2+5000s+10000}$$

is unstable (two poles in RHP)

poles are: 376.6548, 15.1031, -1.7579

$$P^{-1}(s)T(s) = \frac{\frac{10(0.5s+1)}{s}}{1+\frac{10(0.5s+1)(-0.1s+1)}{s(0.01s+1)(0.1s+1)}} = \frac{10(0.5s+1)(0.01s+1)(0.1s+1)}{s(0.01s+1)(0.1s+1)+10(0.5s+1)(-0.1s+1)}$$

$$= \frac{0.005s^3+0.56s^2+6.1s+10}{0.001s^3-0.39s^2+5s+10} = \frac{5s^3+560^2+6100s+10000}{s^3-390s^2+5000s+10000}$$

is unstable.

$$P(s)S(s) = P(s)\left[1-T(s)\right] = \frac{-0.1s+1}{(0.01s+1)(0.1s+1)}\left[1-\frac{-500s^2+4000s+10000)}{s^3-390s^2+5000s+10000}\right]$$

$$= \frac{-0.1s+1}{(0.01s+1)(0.1s+1)}\left(\frac{s^3+110s^2+1000s}{s^3-390s^2+5000s+10000}\right)$$

is unstable (two poles in RHP and proper). Therefore the feedback system is unstable. (note that anyone of the tests above is sufficient to conclude)

Using Theorem II:

We have to show that either $T(s)$ or $S(s)$ is stable, and that no pole-zero cancellation occurs in the closed RHP in forming the loop gain. The latter condition holds, but

$$T(s) = \frac{-500(s^2-8s-20)}{s^3-390s^2+5000s+10000}$$

is unstable (poles are: 376.6548, 15.1031, -1.7579: two poles in RHP.)

Therefore the feedback system is unstable.

Using Theorem III:

We have to show that the closed-loop poles, i.e., the zeros of the characteristic polynomial $p(s)$, are all in the open LHP. The plant and the controller are already expressed as ratios of coprime polynomials.

$$
\begin{aligned}
p(s) &= n_K n_P + d_K d_P \\
&= 0.001s^3 - 0.39s^2 + 5s + 10 \\
&= 0.001(s^3 - 390s^2 + 5000s + 10000) \\
&= 0.001(s - 376.7)(s - 15.1)(s + 1.8)
\end{aligned}
$$

Two of the closed-loop poles lie in the RHP and therefore the feedback system is unstable.

Using Theorem IV:

We have to show that $1 + K(s)P(s)$ has no zero in the closed RHP, and that no pole-zero cancellation occurs in the closed RHP in forming the loop gain. The latter condition obviously holds, and

$$
\begin{aligned}
1 + K(s)P(s) = \frac{1}{S(s)} &= \frac{s^3 - 390s^2 + 5000s + 10000}{s^3 + 110s^2 + 1000s} \\
&= \frac{(s - 376.7)(s - 15.1)(s + 1.8)}{s^3 + 110s^2 + 1000s}
\end{aligned}
$$

has two closed RHP zeros. Therefore, the feedback system is unstable.

Problem 11.6

Consider the LTI unity feedback regulator of Figure 11.9.

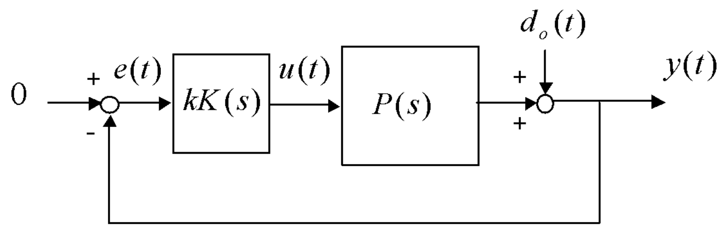


Figure 11.9: Feedback regulator of Problem 11.6.

The transfer functions in this regulator are $P(s)=\dfrac{(s^2-4s+4)}{(s-1)(s^2+\sqrt{2}s+1)}$, $K(s)=\dfrac{1}{s+2}$ and $k\in[0,+\infty)$. Use properties of the root locus to sketch it. Check your sketch using MATLAB (*rlocus* command).

Problem 11.7

Consider the LTI feedback control system shown in Figure 11.10, where $G_a(s)=\dfrac{10}{(s+3)}$, $K(s)=\dfrac{100(2s+1)}{(s+1)}$ and the plant model P is $P(s)=\dfrac{s+2}{s}$.

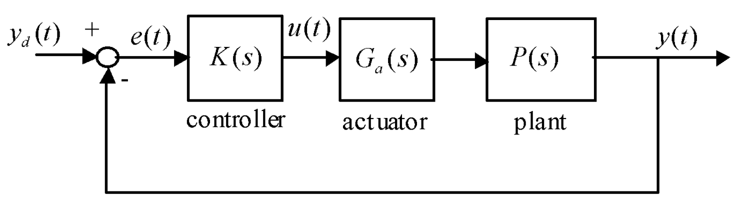


Figure 11.10: Feedback control system of Problem 11.7.

(a) Compute the loop gain $L(s)$, and the closed-loop characteristic polynomial $p(s)$. Is the closed-loop system stable?

Answer:

$$L(s) = K(s)G_a(s)P(s) = \frac{100(2s+1)}{(s+1)}\frac{10}{(s+3)}\frac{s+2}{s} = \frac{1000(2s+1)(s+2)}{s(s+1)(s+3)},$$

$$\begin{aligned} p(s) &= 1000(2s+1)(s+2) + s(s+1)(s+3) \\ &= 1000(2s^2+5s+2) + (s^3+4s^2+3s) \\ &= 2000s^2 + 5000s + 2000 + s^3 + 4s^2 + 3s \\ &= s^3 + 2004s^2 + 5003s + 2000 \end{aligned}$$

Stability is checked by ensuring that all the zeros of $p(s)$ are in the open LHP (using Matlab):

```
roots([1 2004 5002 2000])

ans =

  1.0e+003 *

  -2.00150087506221

  -0.00199933325951

  -0.00049979167827
```

Therefore, the closed-loop system is stable.

(b) Find the sensitivity function $S(s)$ and the complementary sensitivity function $T(s)$.

Answer:

The sensitivity is:

$$S(s)=\frac{1}{1+L(s)}=\frac{1}{1+\frac{1000(2s+1)(s+2)}{s(s+1)(s+3)}}=\frac{s(s+1)(s+3)}{s(s+1)(s+3)+1000(2s+1)(s+2)}$$
$$=\frac{s(s+1)(s+3)}{(s^2+s)(s+3)+1000(2s^2+5s+2)}=\frac{s(s+1)(s+3)}{s^3+4s^2+3s+2000s^2+5000s+20000}$$
$$=\frac{s(s+1)(s+3)}{s^3+2004s^2+5003s+2000}$$

And the transmission is

$$T(s)=1-\frac{s(s+1)(s+3)}{s^3+2004s^2+5003s+2000}$$
$$=\frac{s^3+2004s^2+5003s+2000-s(s+1)(s+3)}{s^3+2004s^2+5003s+2000}.$$
$$=\frac{2000s^2+5000s+2000}{s^3+2004s^2+5003s+2000}$$

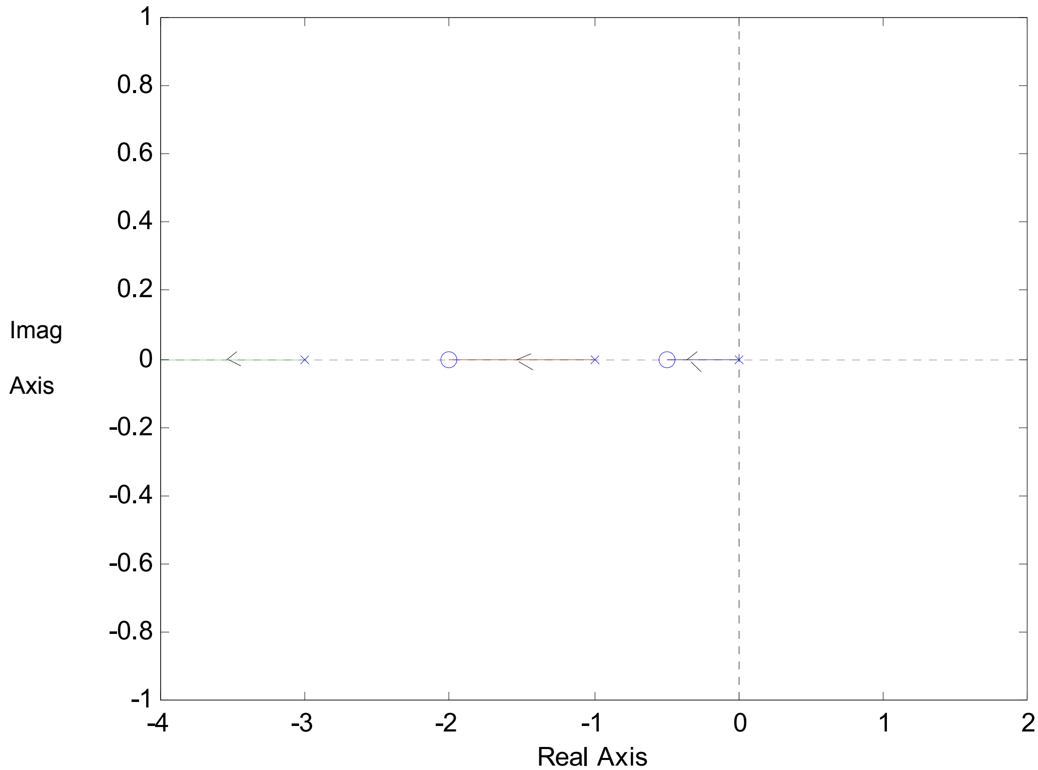
(c) Find the steady-state error of the closed-loop step response $y(t)$,i.e., for the step reference $y_d(t)=u(t)$.

Answer:

The Laplace transform of the error signal is $\hat{e}(s)=\frac{1}{s}S(s)$, and from the final value theorem, we have:

$$\lim_{t\to+\infty}e(t)=S(0)=\frac{0(0+1)(0+3)}{(0)^3+2004(0)^2+5003(0)+2000}=0$$

(d) Assume that the real parameter k is an additional gain on the controller, i.e., the controller is $kK(s)$. Sketch the root locus for the real parameter varying in $k\in[0,+\infty)$.



Problem 11.8

Sketch the Nyquist plots of the following loop gains. Assess closed-loop stability (use the critical point -1.)

(a) $L(s) = \dfrac{1}{(s+1)^2}$

(b) $L(s) = \dfrac{1}{(s+1)(s-1)}$

Problem 11.9

We wish to analyze the stability of the tracking control system in Figure 11.11, where $P(s) = \dfrac{-0.1s+1}{(s+1)(10s+1)}$ and $K(s) = \dfrac{(s+1)}{s}$.

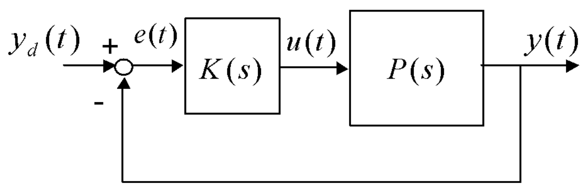
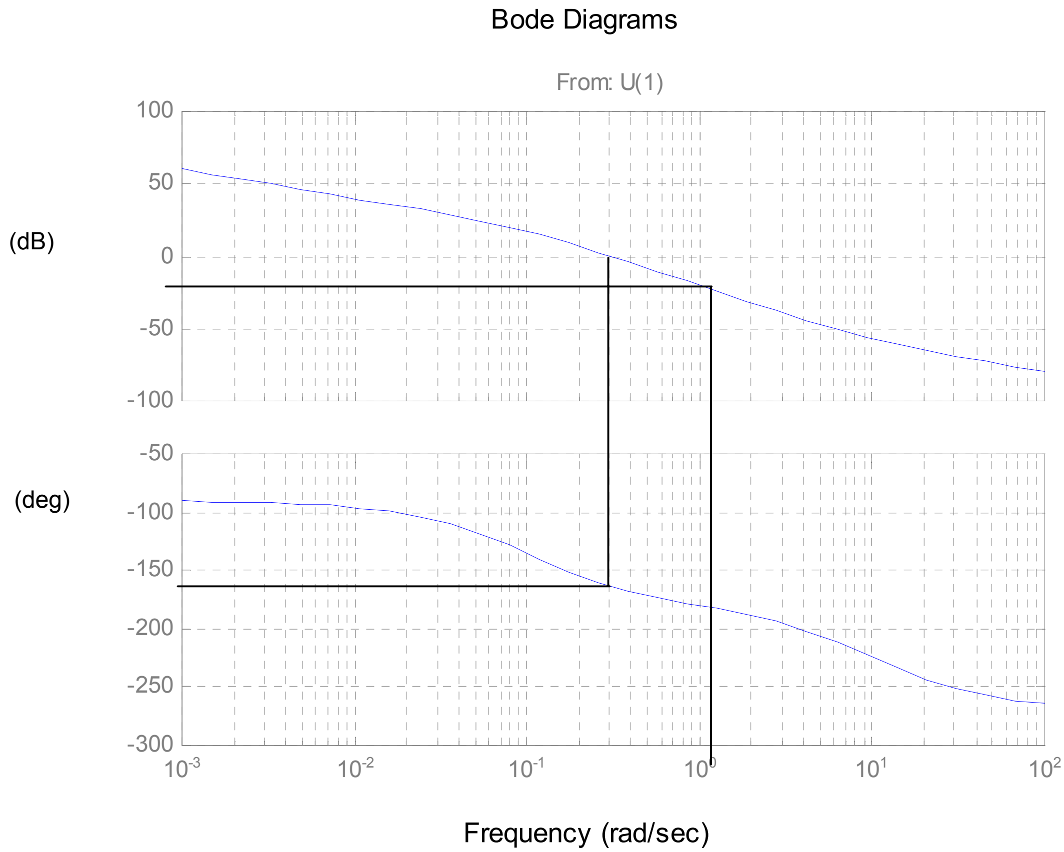


Figure 11.11: Feedback control system of Problem 11.9.

Hand sketch the Bode plot in dB's (magnitude) and degrees (phase) of the loop gain. You can use Matlab to check if your sketch is right. Find the crossover frequency ω_{co} and the frequency where the phase is -180°: $\omega_{-\pi}$. Assess the stability robustness by finding the gain margin k_m and phase margin ϕ_m of the system. Compute the minimum time delay in the plant that would cause instability.

Answer:

$$L(s) = \frac{-0.1s+1}{s(10s+1)},$$



$\omega_{-\pi} \approx 1$ rd/s so the gain margin $k_m \approx 20\text{dB}$, i.e., $k_m \approx 10$.

phase margin $\phi_m \cong 15^\circ$ at $\omega_{co} \cong 0.3\text{rd/s}$.

Minimum time delay τ in the plant that would cause instability:

$$\omega_{co}\tau = \frac{\pi}{180}\phi_m$$
$$\Rightarrow \tau = \frac{\pi}{180\omega_{co}}\phi_m = \frac{15\pi}{180 \cdot 0.3} = 0.873\,\text{s}$$

Problem 11.10

Sketch of the Nyquist plot of the loop gain $L(s) = \frac{s+2}{(s+1)(10s+1)}$. Identify the frequencies where the locus of $L(j\omega)$ crosses the unit circle and the real axis (if it does.)

Problem 11.11

Space rendez-vous

Consider the spacecraft shown in Figure 11.12 which has to maneuver in order to dock on a space station.

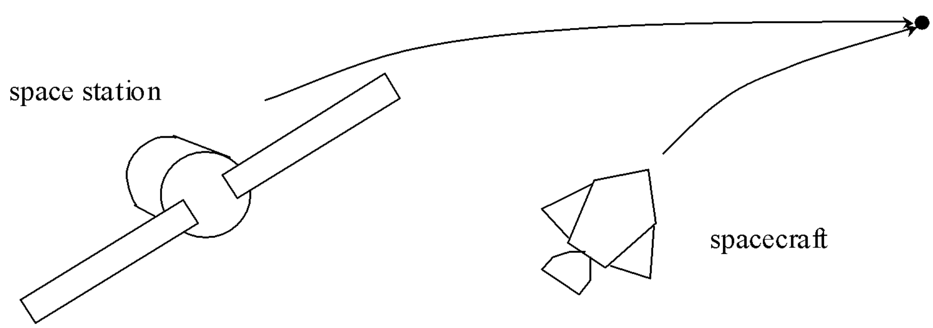


Figure 11.12: Spacecraft docking on a space station in Problem 11.11.

For simplicity, we consider the one-dimensional case where the state of each vehicle consists of its position and velocity along a single axis. Assume that the space station moves autonomously according to the state equation:

$$\dot{x}_s(t) = A_s x_s(t),$$

where $x_s = \begin{bmatrix} z_s \\ \dot{z}_s \end{bmatrix}$, and $A_s = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, and the spacecraft's equation of motion is:

$$\dot{x}_c(t) = A_c x_c(t) + B_c u_c(t),$$

where $x_c = \begin{bmatrix} z_c \\ \dot{z}_c \end{bmatrix}$, $u_c(t)$ is the thrust, $A_c = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $B_c = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}$.

(a) Write down the state-space system of the state error $e := x_c - x_s$ which describes the evolution of the difference in position and velocity between the spacecraft and the space station. The output is the difference in position.

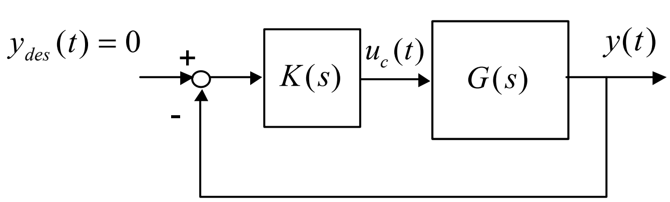
Answer:

Let $e := x_c - x_s = \begin{bmatrix} z_c \\ \dot{z}_c \end{bmatrix} - \begin{bmatrix} z_s \\ \dot{z}_s \end{bmatrix}$, so that:

$$\begin{aligned} \dot{e}(t) &= A_c(x_c(t) - x_s(t)) + B_c u_c(t) \\ &= A_c e(t) + B_c u_c(t) \\ &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} e(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u_c(t) \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} e(t) \end{aligned}$$

(b) A controller is implemented in a unity feedback control system to drive the position difference to zero for automatic docking. The controller is given by:

$$K(s) = \frac{100(s+1)}{0.01s+1}, \ \ \mathrm{Re}\{s\} > -100$$



Find $G(s)$ and assess the stability of this feedback control system (hint: one of the closed-loop poles is at -10.)

Answer:

$$
\begin{aligned}
G(s) &= C_e(sI - A_e)^{-1}B_e + D_e \\
&= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} \\
&= \frac{0.1}{s^2}, \ \mathrm{Re}\{s\} > 0
\end{aligned}
$$

It is a double integrator, which is to be expected with Newton's law applied to free-floating bodies. Closed-loop characteristic polynomial with coprime numerators and denominators:

$$
\begin{aligned}
p(s) &= n_G n_K + d_G d_K = 0.1(100s + 100) + s^2(0.01s + 1) \\
&= 0.01s^3 + s^2 + 10s + 10 = 0.01(s^3 + 100s^2 + 1000s + 1000)
\end{aligned}.
$$

Thus,

$$
\begin{aligned}
s^3 + 100s^2 + 1000s + 1000 &= (s + a)(s + b)(s + 10) \\
&= s^3 + (10 + a + b)s^2 + (10a + 10b + ab)s + 10ab
\end{aligned}.
$$

By identifying coefficients, we find three equations in two unknowns (one is redundant):

$$
\begin{aligned}
10 + a + b &= 100 \\
10a + 10b + ab &= 1000 \\
10ab &= 1000
\end{aligned}
$$

We combine the first and third equations to find a quadratic equation:

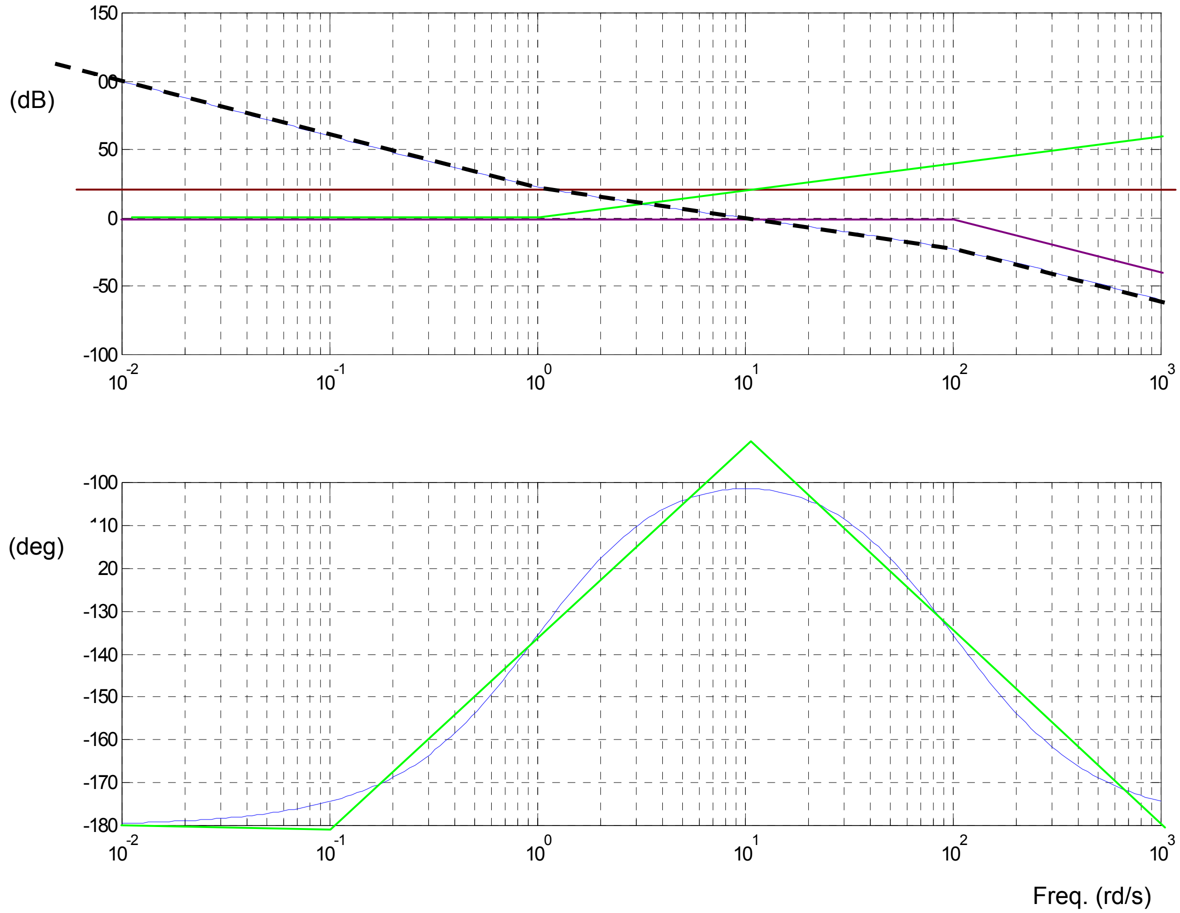
$$a^2 - 90a + 100 = 0$$
$$\Rightarrow a_1 = 45 + 10\sqrt{4.5^2 - 1} = 88.8748$$
$$a_2 = 45 - 10\sqrt{4.5^2 - 1} = 1.1252$$

It turns out that for the choice $a = 88.8748 \Rightarrow b = 1.1252$ and for the choice $a = 1.1252 \Rightarrow b = 88.8748$. Therefore, the three closed-loop poles are at $-10, -1.1252, -88.8748$, and the closed-loop system is stable.

(c) Find the loop gain, sketch its Bode plot, and compute the phase margin of the closed-loop system. Assuming for the moment that the controller would be implemented on earth, what would be the longest communication delay that would not destabilize the automatic docking system?

Answer:

Loop gain: $L(s) = \dfrac{10(s+1)}{s^2(0.01s+1)}$. Its Bode plot is shown below, with the broken line approximation superimposed on the actual plot..



The crossover frequency is approximately $\omega_{co} = 10\text{rd/s}$. The phase margin is given by:

$$\angle L(j\omega_{co}) \cong -102^{\circ} \Rightarrow \phi_m = 78^{\circ}.$$

From the broken line approximation of the Bode plot, we find a rough value fo the phase margin of

$$\angle L(j\omega_{co}) \cong -90^{\circ} \Rightarrow \phi_m = 90^{\circ}.$$

Using the latter, we compute the maximum time-delay that can be tolerated:

$$\omega_{co}\tau = \frac{\pi}{180}\phi_m$$
$$\Rightarrow \tau = \frac{\pi}{180\omega_{co}}\phi_m = \frac{90\pi}{180\cdot 10} = 0.1571\text{s}$$

(d) Compute the sensitivity function of the system and give the steady-state error to a unit step disturbance on the output.

Answer:

$$S(s) = \frac{1}{1+L(s)} = \frac{1}{1+\dfrac{10(s+1)}{s^2(0.01s+1)}} = \frac{s^2(0.01s+1)}{0.01s^3+s^2+10s+10}$$

The Laplace transform of the error signal is $\hat{e}(s) = \frac{1}{s}S(s)$, and from the final value theorem, we have $\lim_{t\to+\infty} e(t) = S(0) = \frac{0}{10} = 0$.